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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

V394

## A THREE-DIMENSIONAL NONSINGULAR SIMULATION OF RIGID MANIPULATORS

by

Robert M. Verbos

September 1988

Thesis Advisor:

D. Smith

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**A Three-Dimensional Nonsingular Simulation  
of Rigid Manipulators**

by

Robert M. Verbos  
Lieutenant, United States Navy  
B.E.E., Villanova University, 1981

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

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September 1988

## **ABSTRACT**

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## LIST OF SYMBOLS AND ABBREVIATIONS

A	Sine Wave Input Torque Amplitude
AA	Acceleration of Point A
AB	Acceleration of Point B
<u>AG1</u>	The Acceleration Vector of the Center of Gravity for Link 1
<u>AG2</u>	Same as AG1 but for Link 2
<u>AG3</u>	Same as AG1 but for Link 3
AMP	Magnitude of the Unit Vector of the Directional Cosines
AX0	Linear Acceleration of Joint 0 in the X Direction
AY0	Linear Acceleration of Joint 0 in the Y Direction
AZ0	Linear Acceleration of Joint 0 in the Z Direction
AX1, AY1, AZ1	Linear Acceleration of Link 1 in the Respective Direction
AX2, AY2, AZ2	Linear Acceleration of Link 2 in the Respective Direction
AX3, AY3, AZ3	Linear Acceleration of Link 3 in the Respective Direction
CPROD	Subroutine to Calculate a Cross Product of Two Vectors
DEGRA	DSL Conversion From Degrees to Radians



DRCANX(1), DRCANX(2), DRCANX(3)	Direction Cosine Angle in Degrees in the Fixed Coordinate System From the X Axis for Links 1-3, Respectively
DRCANY(1), DRCANY(2), DRCANY(3)	Same as above except from the Y Axis
DRCANZ(1), DRCANZ(2), DRCANZ(3)	Same as above except from the Z Axis
DRCRAX(1) DRCRAX(2) DRCRAX(3)	Direction Cosine Angle in Radians in the Fixed Coordinate System from the X Axis for Links 1-3, Respectively
DRCRAY(1), DRCRAY(2), DRCRAY(3)	Same as above except from the Y Axis
DRCRAZ(1), DRCRAZ(2), DRCRAZ(3)	Same as above except from the Z Axis
DRCSX(1), DRCSX(2), DRCSX(3)	The Argument of the Direction Cosine Angle in the X Direction for Links 1-3, Respectively
DRCSY(1), DRCSY(2), DRCSY(3)	Same as above except in the Y Direction
DRCSZ(1), DRCSZ(2), DRCSZ(3)	Same as above except in the Z Direction
<u>DQ</u>	A 27 x 1 Column Matrix Obtained by Multiplying MATA by MATB
FX0	Computed Force in the X Direction at Joint 0
FY0	Computed Force in the Y Direction at Joint 0
FZ0	Computed Force in the Z Direction at Joint 0
FX1, FY1, FZ1	Same as above except at Joint 1

FX2, FY2, FZ2	Same as above except at Joint 2
G	Gravitational Constant
I	Counter
IA	Row Dimension of MATA and MATB Used in IMSL Routine LEQT2F
IER	Error Parameter Used in IMSL Routine LEQT2F
IGDT	Accuracy Test Used in IMSL Routine LEQT2F
IMAT	A 3 x 3 x 3 Matrix which contains the Global Inertias of Links 1, 2, and 3
IXX	A 3 x 2 Matrix of Moment of Inertia for the Two-Element Composite Body of Links 1-3 About the X Axis
IYY	Same as IXX but About the Y Axis
IZZ	Same as IXX but About the Z Axis
IXZ	A 3 x 2 Matrix of Product of Inertia for the Two-Element Composite Body of Links 1-3 About the XZ Coordinate Axis
IXY	Same as IXZ but for the XY Axis
IYZ	Same as IXZ but for the YZ Axis
IXXT	Total Moment of Inertia of Links 1-3 About the X Axis
IYYT	Same as IXXT but About Y Axis
IZZT	Same as IXXT but About Z Axis
IXZT	Same as IXXT but About XZ Axis
IXYT	Same as IXXT but About XY Axis
IYZT	Same as IXXT but About YZ Axis
JX0	Location of Joint 0 in the X Direction

JY0	Location of Joint 0 in the Y Direction
JZ0	Location of Joint 0 in the Z Direction
JX1, JY1, JZ1	Same as above except for Joint 1
JX2, JY2, JZ2	Same as above except for Joint 2
<u>L</u>	A 3 x 2 Matrix, Distance from Center of Gravity of the Link to the Center of Mass at the Link End
<u>LCOGX</u>	A 1 x 3 Matrix, Location of the Link Center of Gravity in the X Direction
<u>LCOGY</u>	Same as LCOGX but for the Y Direction
<u>LCOGZ</u>	Same as LCOGX but for the Z Direction
LEVELQ	Error Set for Routine LEQT2F
LEVLDQ	Error Set for Routine LEQT2F
<u>LIMAT</u>	A 3 x 3 x 3 Matrix which Contains the Local Inertias of Links 1, 2, and 3
LNCOG1	The Length of the Vector from Joint 0 to the COG of Link 1
LNCOG2	The Length of the Vector from Joint 1 to the COG of Link 2
LNCOG3	The Length of the Vector from Joint 2 to the COG of Link 3
M	Number of the Right-Hand Side Used in LEQT2F
<u>MASS</u>	A 3 x 2 Matrix of the Masses of the Elements of the Link Composite
M1	Total Mass of Link 1
M2	Total Mass of Link 2
M3	Total Mass of Link 3

<u>MATA</u>	A 27 x 27 Matrix Consisting of the Coefficients of the Unknown Variables
<u>MATB</u>	A 27 x 1 Vector Consisting of the Coefficients of Knowns on Input, and the Solution from the IMSL Routine LEQT2F on Output
<u>MATC</u>	A 27 x 1 Column Vector which Contains the Results of the Multiplication MATA * MATD
<u>MATD</u>	A 27 x 1 Column Vector which Contains the Knowns in the Simulation Validation
MI	X Component, Cross-Product from Subroutine CPROD
MJ	Y Component, Cross-Product from Subroutine CPROD
MK	Z Component, Cross-Product from Subroutine CPROD
MI1, MJ1, MK1	Cross-Product of WD x R used in the Validation Program in the the Calculation of Acceleration
MI2, MJ2, MK2	Cross Product of W x W x R used in the Validation Program to Calculate Acceleration
MIC0, MJC0, MKC0	Cross-Product of W1 x W1 x RBG1 in the X, Y, and Z Direction
MIC1, MJC1, MKC1	Cross-Product of W1 x W1 x RAG1 in the X, Y, and Z Direction
MIC2, MJC2, MKC2	Cross-Product of W2 x W2 x RBG2 in the X, Y, and Z Direction
MIC3, MJC3, MKC3	Cross-Product of W2 x W2 x RAG2 in the X, Y and Z Direction
MIC4, MJC4, MKC4	Cross-Product of W3 x W3 x RBG3 in the X, Y, and Z Direction
N	Order of MATA and Number of Rows in MATB
P	Phase Angle of Sin Wave
RADEG	DSL Conversion from Radians to Degrees



<u>RX</u>	A 3 x 2 Matrix of the Distance from the COG of the Link to the Center of the Link Masses in the X Direction
<u>RY</u>	Same as RX except in Y direction
<u>RZ</u>	Same as RX except in Z direction
<u>RAG1</u>	A 1 x 3 Vector of the Distance of Point A to the COG of Link 1 in the X, Y, and Z Direction
<u>RAG2</u>	Same as RAG1 except for Link 2
<u>RAG3</u>	Same as RAG1 except for Link 3
<u>RBG1</u>	A 1 x 3 Vector of the Distance of Point B to the COG of Link 1 in the X, Y, and Z Direction
<u>RBG2</u>	Same as RBG1 except for Link 2
<u>RBG3</u>	Same as RBG2 except for Link 3
T0X, T0Y, T0Z	Torque at Joint 0 in the X, Y, and Z Direction
T1FNC, T2FNC	Torque at Joints 1-2 beyond the Gravitational Torque
T1X, T1Y, T1Z	Torque at Joint 1 in the X, Y, and Z Direction
T2X, T2Y, T2Z	Torque at Joint 2 in the X, Y, and Z Direction
TG1, TG2	Gravitation Torque at Joints 1-2
TIPX, TIPY, TIPZ	Position of the End Effector
<u>TMAT</u>	A 3 x 3 Transformation Matrix which Transforms the Local Coordinates to Global
<u>TMATTR</u>	A 3 x 3 Matrix which is the Transpose of TMAT
<u>TMPMAT</u>	A Temporary 3 x 3 Matrix used in Matrix Multiplication
<u>VECTA</u> , <u>VECTB</u>	A 1 x 3 Vector Used in the Subroutine CPROD
W	Frequency of Sine Wave

WG1, WG2, WG3	Weight of Links 1-3
<u>W1</u> , <u>W2</u> , <u>W3</u>	A 1 x 3 Vector of the Angular Velocity of Links 1-3
WDX, WDY, WDZ	Angular Acceleration of Links 1-3 in the X, Y, and Z Direction
WKAREA	Work Area used in the IMSL Routine LEQT2F
X1, X2, X3	Initial Location of the COG of Link 1-3 in the X Direction
Y1, Y2, Y3	Initial Location of the COG of Link 1-3 in the Y Direction
Z1, Z2, Z3	Initial Location of the COG of Link 1-3 in the Z Direction

## **ACKNOWLEDGMENTS**

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## I. INTRODUCTION

The study of robotics is a fairly new area in the fields of science and engineering which has fundamentals that can be traced back to many basic disciplines, such as electrical engineering, mechanical engineering, and computer science. The mechanical engineering interest in the study of robotic manipulators can be traced back to as early as the 1940s, although the first industrial robots were not introduced until the late 1950s and early 1960s [Ref. 1]. Mechanical engineers were mostly interested in two areas of robotics: the control of the linkage and the study of its motion. While these two areas are interrelated, this thesis focuses on the second: the study of the motion of the linkage.

The study of robot motion (dynamics) is further divided into the study of arm dynamics and robot arm kinematics. Both these divisions are subdivided into a direct and an inverse problem. The direct dynamics problem is the calculation of link state variables such as acceleration, velocity, and joint angles from the known joint torques. The inverse dynamic problem assumes state variables are known and the joint torques are to be calculated. In the kinematics, the direct problem is the determination of the end effector position and its orientation from a given set of link geometries and joint angles. The inverse problem is the calculation of the link geometries and joint angles for a given end effector position.



Various methods to solve the inverse kinematic problem use matrix transformations, inversion, and multiplication, and as such are subjected to singularity problems [Ref. 2]. Singularity occurs when two successive links are aligned (i.e., the angle between the links is 0 or 180 degrees). When a singularity occurs, the matrices involved are also singular and not invertible.

A method to avoid the singularity problem was developed at the Naval Postgraduate School using Newtonian dynamics in terms of a globally fixed coordinate system [Ref. 3]. This method treats each link as a free body with the forces and moments applied at the joints and the use of Newton's law to derive the equations of motion. In this thesis, the development of the free body analysis approach to rigid manipulators dynamics is continued and extended to three dimensions and including gravitational effects.

## II. PROBLEM STATEMENT

Theoretical dynamic approaches are well developed and have been extensively used with simulations. However, in manipulator studies, these accepted methods all have a singularity that arises when two successive links are aligned. This does not allow the motion to be simulated [Ref. 3]. This shortfall cannot be tolerated in the modelling of an actual robot or robotic manipulator because it is of the utmost importance to accurately know the arm position at all times (for real-time control purposes) [Ref. 2]. Therefore, the problem directed by this thesis research project was:

Continue the development of the Newton Euler free-body approach for a three-link manipulator extended to three-dimensional motion and including gravitational effects. In addition, compare simulation results with those of an actual robotic arm. [Ref. 4]

### III. BACKGROUND

#### A. THEORY

The basic theory for the Newton Euler free-body analysis to overcome the singularity problems of conventional robot dynamics was developed by Sadrettin Altinok [Ref. 3]. The theory of treating each link of the manipulator as a free body with respect to a global coordinate system leads to the following 27 equations for a three-link manipulator.

$$FX0 + M1*AX1 - FX1 = 0 \quad (1)$$

$$FY0 + M1*AY1 - FY1 = 0 \quad (2)$$

$$FZ0 + M1*AZ1 - FZ1 = -WG1 \quad (3)$$

$$AX1 + RB1Z*WD1Y - RB1X*WD1Z = AX0 - MIC0 \quad (4)$$

where MIC0 = (X component of)

$$\underline{WD1} \times \underline{RBG1} + \underline{W1} \times (\underline{W1} \times \underline{RBG1}) \quad (4a)$$

$$AY1 - RB1Z*WD1X + RB1X*WD1Z = AY0 - MJC0 \quad (5)$$

where MJC0 = (Y component of equation 4a).

$$AZ1 + RBY1*WD1X - RB1Z*WD1Y = AZ0 - MKC0 \quad (6)$$

where MKC0 = (Z component of equation 4a).

$$\begin{aligned} RB1Z*FY0 - RB1Y*FZ0 - IXXT1*WD1X + IXYT1*WD1Y + \\ IXZT1*WD1Z - RA1Z*FY1 + RA1Y*FZ1 = T1X - T0X \end{aligned} \quad (7)$$

$$\begin{aligned} -RB1Z*FX0 + RB1X*FZ0 + IXYT1*WD1X - IYYT1*WD1Y + \\ IYZT1*WD1Z + RA1Z*FX1 - RA1X*FZ1 = T1Y - T0Y \end{aligned} \quad (8)$$

$$\begin{aligned} RB1Y*FX0 - RB1Z*FY0 + IXZT1*WD1X + IYZT1*WD1Y - \\ IZZT1*WD1Z - RA1Y*FX1 + RA1X*FY1 = T1Z - T0Z \end{aligned} \quad (9)$$

$$FX1 + M2*AX2 - FX2 = 0 \quad (10)$$

$$FY1 + M2*AY2 - FY2 = 0 \quad (11)$$

$$FZ1 + M2*AZ2 - FZ2 = -WG2 \quad (12)$$

$$\begin{aligned} -AX1 - RA1Z*WD1Y + RA1Y*WD1Z + AX2 + RB2Z*WD2Y - \\ RB2Y*WD2Z = MIC1 - MIC2 \end{aligned} \quad (13)$$

where MIC1 = (X component of)

$$\underline{WD1} \times \underline{RA1} + \underline{W1} \times (\underline{W1} \times \underline{RA1}) \quad (13a)$$

where MIC2 = (X component of)

$$\underline{WD2} \times \underline{RB2} + \underline{W2} \times (\underline{W2} \times \underline{RB2}) \quad (13b)$$

$$\begin{aligned} -AY1 + RA1Z*WD1X - RA1X*WD1Z + AY2 - RB2Z*WD2X + \\ RB2X*WD2Z = MJC1 - MJC2 \end{aligned} \quad (14)$$

where MJC1 = (Y component of equation 13a) and where MJC2 = (Y component of equation 13b).

$$\begin{aligned} -AZ1 - RA1Y*WD1X + RA1X*WD1Y + AZ2 + RB2Y*WD2X - \\ RB2X*WD2Y = MKC1 - MKC2 \end{aligned} \quad (15)$$

where MKC1 = (Z component of equation 13a) and where MKC2 = (Z component of equation 13b).

$$\begin{aligned} RB2Z*FY1 - RB2Y*FZ1 - IXXT2*WD2X + IXYT2*WD2Y + \\ IXZT2*WD2Z - RA2Z*FY2 + RA2Y*FZ2 = T2X - T1X \end{aligned} \quad (16)$$

$$\begin{aligned} -RB2Z*FX1 + RB2X*FZ1 + IXYT2*WD2X - IYYT2*WD2Y + \\ IYZT2*WD2Z + RA2Z*FX2 - RA2X*FZ2 = T2Y - T1Y \end{aligned} \quad (17)$$

$$\begin{aligned} RB2Y*FX1 + RB2X*FY1 + IXZT2*WD2X + IYZT2*WD2Y - \\ IZZT2*WD2Z - RA2Y*FX2 + RA2X*FY2 = T2Z - T1Z \end{aligned} \quad (18)$$

$$FX2 + M3*AX3 = 0 \quad (19)$$

$$FY2 + M3*AY3 = 0 \quad (20)$$

$$FZ2 + M3*AZ3 = -WG3 \quad (21)$$

$$\begin{aligned} -AX2 - RA2Z*WD2Y + RA2Y*WD2Z + AX3 + RB3Z*WD3Y - \\ RB3Y*WD3Z = MIC3 - MIC4 \end{aligned} \quad (22)$$

where MIC3 = (X component of)

$$\underline{WD2} \times \underline{RA2} + \underline{W2} \times (\underline{W2} \times \underline{RA2}) \quad (22a)$$

where MIC4 = (X component of)

$$\underline{WD3} \times \underline{RB3} + \underline{W3} \times (\underline{W3} \times \underline{RB3}) \quad (22b)$$

$$\begin{aligned} -AY2 + RA2Z*WD2X - RA2X*WD2Z + AY3 - RB3Z*WD3X + \\ RB3X*WD3Z = MJC3 - MJC4 \end{aligned} \quad (23)$$

where MJC3 = (Y component of equation 22a) and where MJC4 = (Y component of equation 22b).

$$\begin{aligned} -AZ2 - RA2Y*WD2X + RA2X*WD2Z + AZ3 + RB3Y*WD3X - \\ RB3X*WD3Y = MKC3 - MKC4 \end{aligned} \quad (24)$$



where MKC3 = (Z component of equation 22a) and where MKC4 = (Z component of equation 22b).

$$\begin{aligned} RB3Z*FY2 - RB3Y*FZ2 - IXXT3*WD3X + IXYT3*WD3Y + \\ IXZT3*WD3Z = - T2X \end{aligned} \quad (25)$$

$$\begin{aligned} -RB3Z*FX2 + RB3X*FZ2 + IXYT3*WD3X - IYYT3*WD3Y + \\ IYZT3*WD3Z = - T2Y \end{aligned} \quad (26)$$

$$\begin{aligned} RB3Y*FX2 - RB3X*FY2 + IXZT3*WD3X + IYZT3*WD3Y - \\ IZZT3*WD3Z = - T2Z \end{aligned} \quad (27)$$

## B. COMPUTATION

The Dynamic Simulation Language (DSL) was used to simulate the direct dynamics problem. A matrix (MATA, a 27 x 27 matrix) was created from the known coefficients of the unknown variables in the previous 27 equations. This was possible due to the assumption that the change in the inertia of the links, the link velocities, and the joint position must be extremely small during a simulated step interval and can thus be treated as being constant during the step. In the direct dynamics problem, a vector (MatB, 27 x 1) was generated from the following components: joint torques, the centers of gravity of the link, and the calculated cross-products of angular velocities (Figure 1). This gave a system of equations in the form of equation 28:





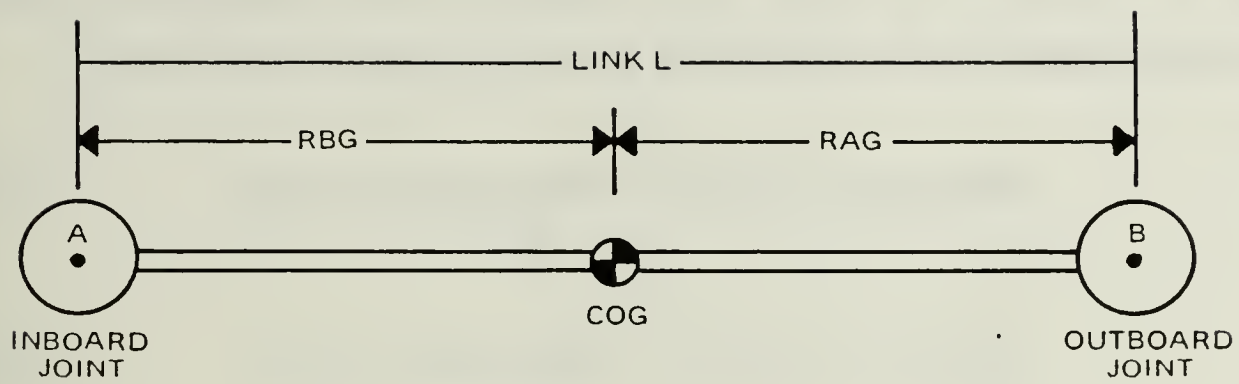
$$\underline{A} * \underline{x} = \underline{b} \quad (28)$$

where  $A$  and  $b$  are known and  $x$  is an unknown vector (which contains the forces at the link ends and the angular and linear accelerations of each link). The IMSL routine LEQT2F (a linear equation solver) was used to obtain the vector  $x$ . [Ref. 3 and Ref. 5]

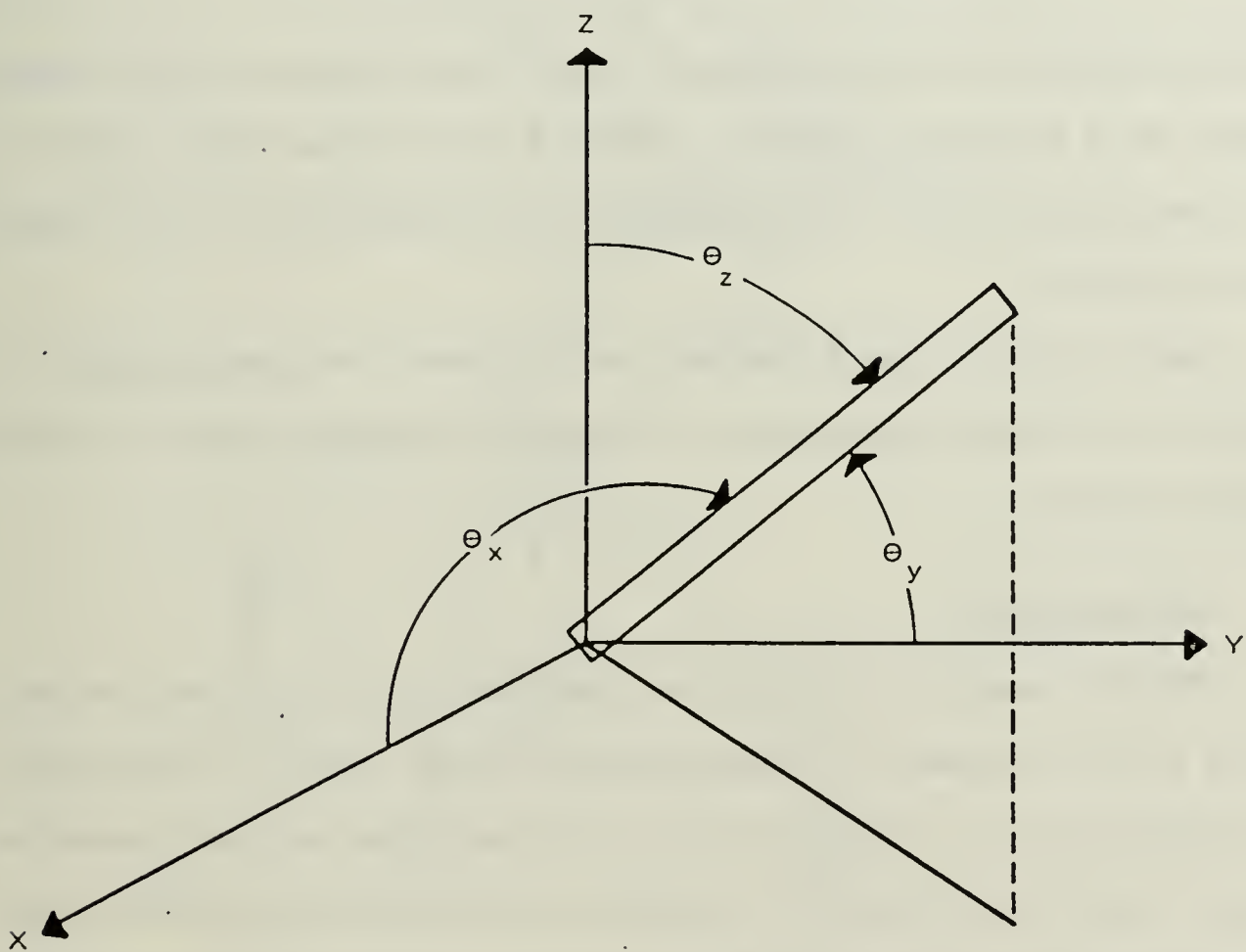
In the inverse problem, MATA was developed the same as in the direct problem as discussed above, but now vector  $x$  was known (from the history of position data) and vector  $b$  was found by straight matrix multiplication of equation 28. [Ref. 3 and Ref. 5]

The previous work used transformation matrix operations to obtain joint orientations [Ref. 3]. This was done because joint orientation cannot be obtained directly from the integration of rotational velocity [Ref. 6 and Ref. 7]. The approach used in this thesis was one of simple geometric relationships. It was reasoned that the linear accelerations were either known or calculated from the matrix operation of LEQT2F (equation 28). These linear accelerations were the linear accelerations of the link center of gravities with respect to the earth fixed (inertial) coordinate system which were integrated to obtain the linear velocities. The velocities were further integrated to obtain the position of the center of gravities.

Knowing the earth fixed coordinates of the centers of gravity, and that each link can be represented by a rigid body [Ref. 8], it was known that the center of gravity must be on the vector from the inboard joint to the end of the link, or next joint (Figure 2). Once the



(a) Center of Gravity



(b) Directional Cosine Angles

Figure 2. **Link Geometries**

vector was known to the center of gravity, the directional cosines were calculated from the following three equations for each link.

$$\text{DRCOSX} = [\text{LCOGX} - \text{JX}(\text{inboard})]/\text{LNCOG} \quad (29)$$

$$\text{DRCOSY} = [\text{LCOGY} - \text{JY}(\text{inboard})]/\text{LNCOG} \quad (30)$$

$$\text{DRCOSZ} = [\text{LCOGZ} - \text{JZ}(\text{inboard})]/\text{LNCOG} \quad (31)$$

where LCOGX is the X location of the CG with respect to the inertia frame; JX is the X location of the inboard joint location with respect to the inertia; and LNCOG is magnitude of the length from the inboard joint to the CG.

Once the directional cosines were known, the same method as used by previous investigators was used to calculate the MATA and MATB matrices.

### **C. CONSTRAINTS**

With the desire to simulate the motion of an actual robotic manipulator and to compare the simulation with data from the manipulator, constraints had to be added into the simulation so the simulation would take into account the the joint mechanical restrictions. In theory, each joint is a one degree of freedom connection. However, in the chosen arm, the zero joint rotates in the Z direction only, while joints one and two rotate about both the X and Y axes (Figure 3). The previously mentioned constraints led to the following equations:

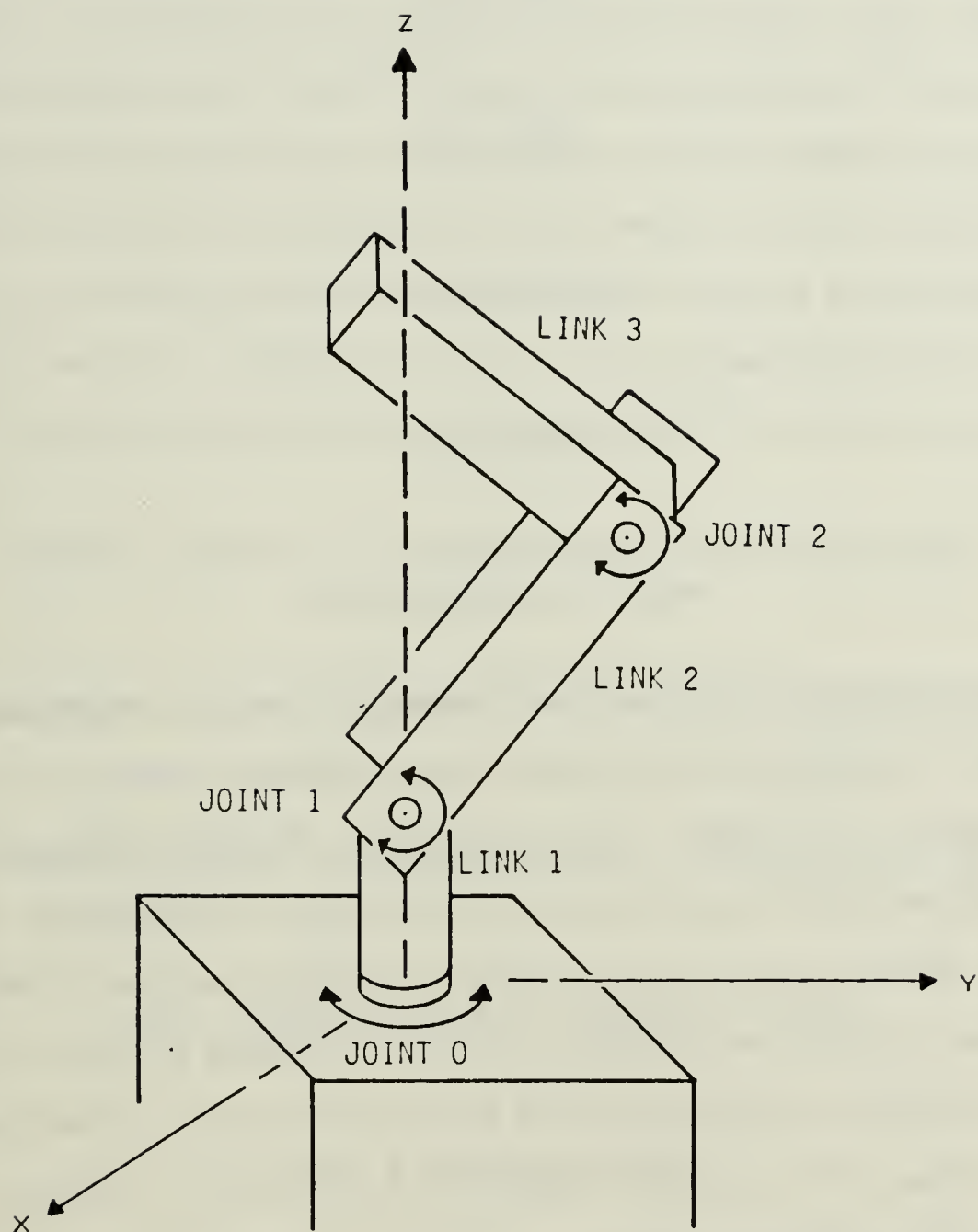


Figure 3. Robot Axis Rotation with Constraints

$$AX1 = 0 \tag{32}$$

$$AY1 = 0 \tag{33}$$

$$AZ1 = 0 \tag{34}$$

$$WDX1 = 0 \tag{35}$$

$$WDY1 = 0 \tag{36}$$

$$WDZ1 = WDZ2 = WDZ3 \tag{37}$$

The concept for entering the constraints into the simulation was developed in Reference 3 and led to the constraint section of the DSL code as seen in Appendix A and Appendix B. The basic concept which was developed was to zero out the row only in MATA, then set the diagonal of that row to one and the corresponding row of MATB to zero. This concept was changed to include zeroing out the row and column of MATA corresponding to the row of vector x, which was to be zero, along with setting the diagonal of the row to 1 and the corresponding row of MATB to zero.

#### **D. GRAVITATIONAL TORQUES**

The simulation of gravitational effects on the arm required that an equalizing torque be applied at the joints to overcome the gravitational



#### D. GRAVITATIONAL TORQUES

The simulation of gravitational effects on the arm required that an equalizing torque be applied at the joints to overcome the gravitational pull (Figure 4). This torque varied as the Z location of the center of gravity of links 2 and 3 varied because the magnitude of the moment arm varied as the link moved. The lever arm which affected the gravitational torque was the projection of the vector from the inboard joint to the link center of gravity onto the XY plane (Figure 5). Knowing this lever arm and the weight of the links, the following equations were used to calculate the equilibrium torque needed to overcome gravity:

$$\text{MARM2A} = \text{SQRT} (\text{LCOGX2} * \text{LCOGX2} + \text{LCOGY2} * \text{LCOGY2}) \quad (38)$$

$$\text{MARM2B} = \text{SQRT} (\text{LCOGX3} * \text{LCOGX3} + \text{LCOGY3} * \text{LCOGY3}) \quad (39)$$

$$\text{MARM3} = \text{SQRT} ((\text{LCOGX3} - \text{JX2}) ** 2 + (\text{LCOGY3} - \text{JY2}) ** 2) \quad (40)$$

$$\text{TG1} = \text{MAR2A} * \text{WG2} + \text{MARM2B} * \text{WG3} \quad (41)$$

$$\text{TG2} = \text{MARM3} * \text{WG3} \quad (42)$$

The equations above represent the magnitude of the required torque which must be resolved into its X and Y components (Figure 6).

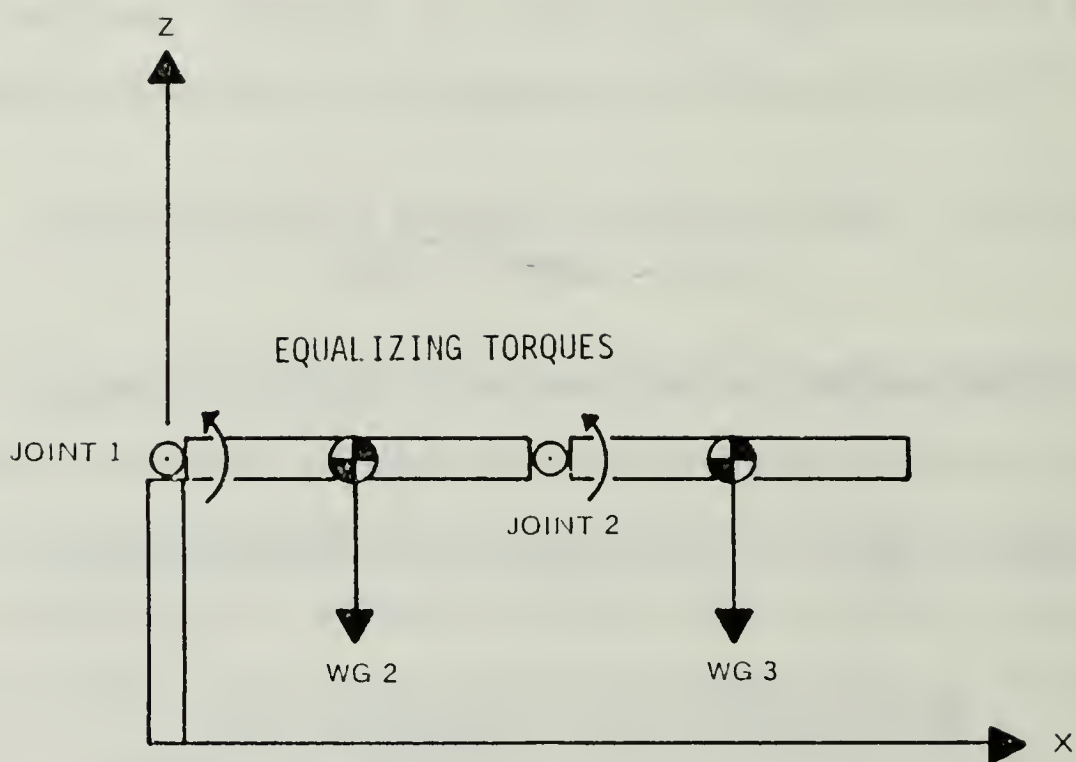


Figure 4. **Gravitational Torques**

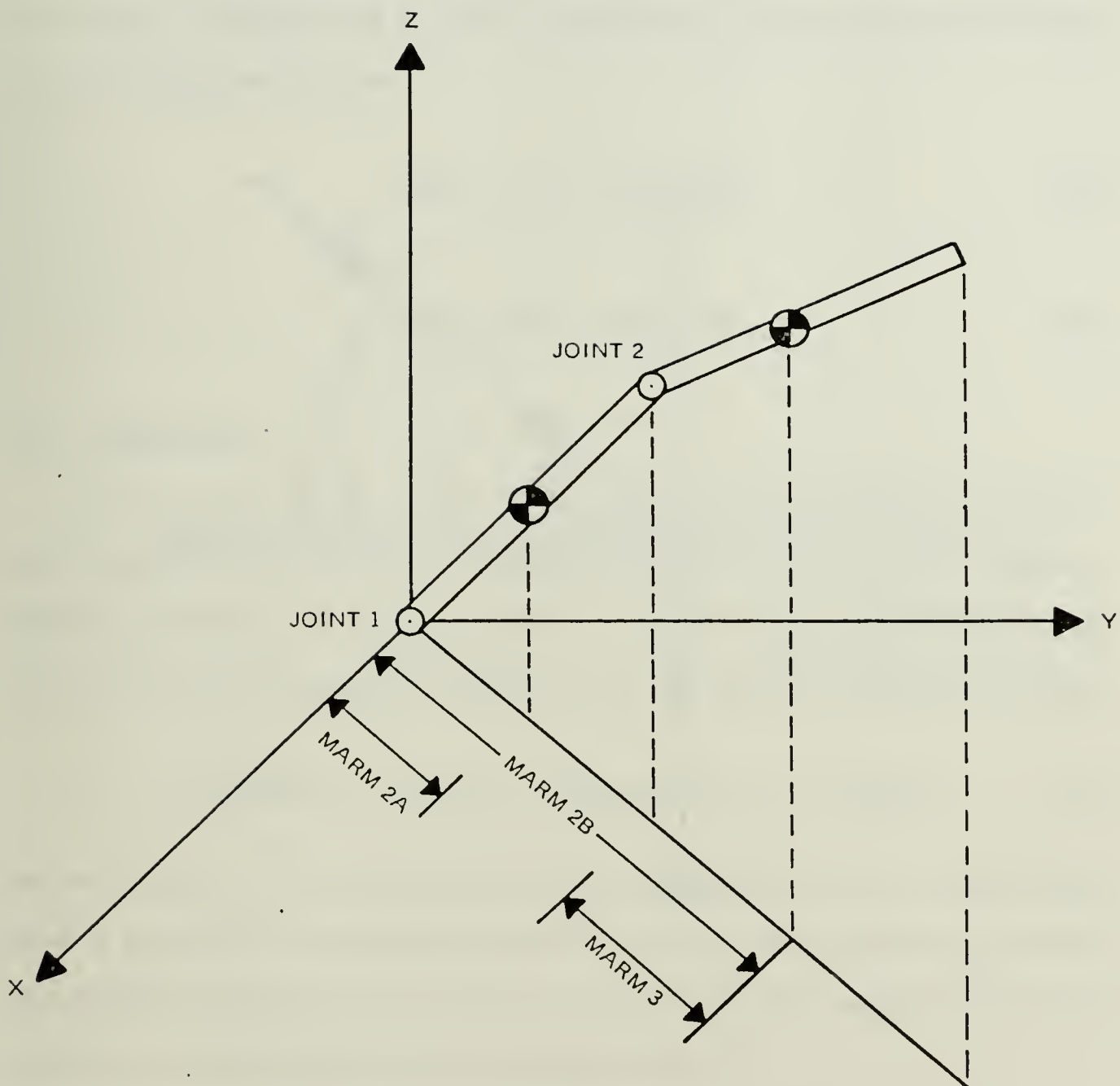


Figure 5. Gravitational Torque Moment Arms



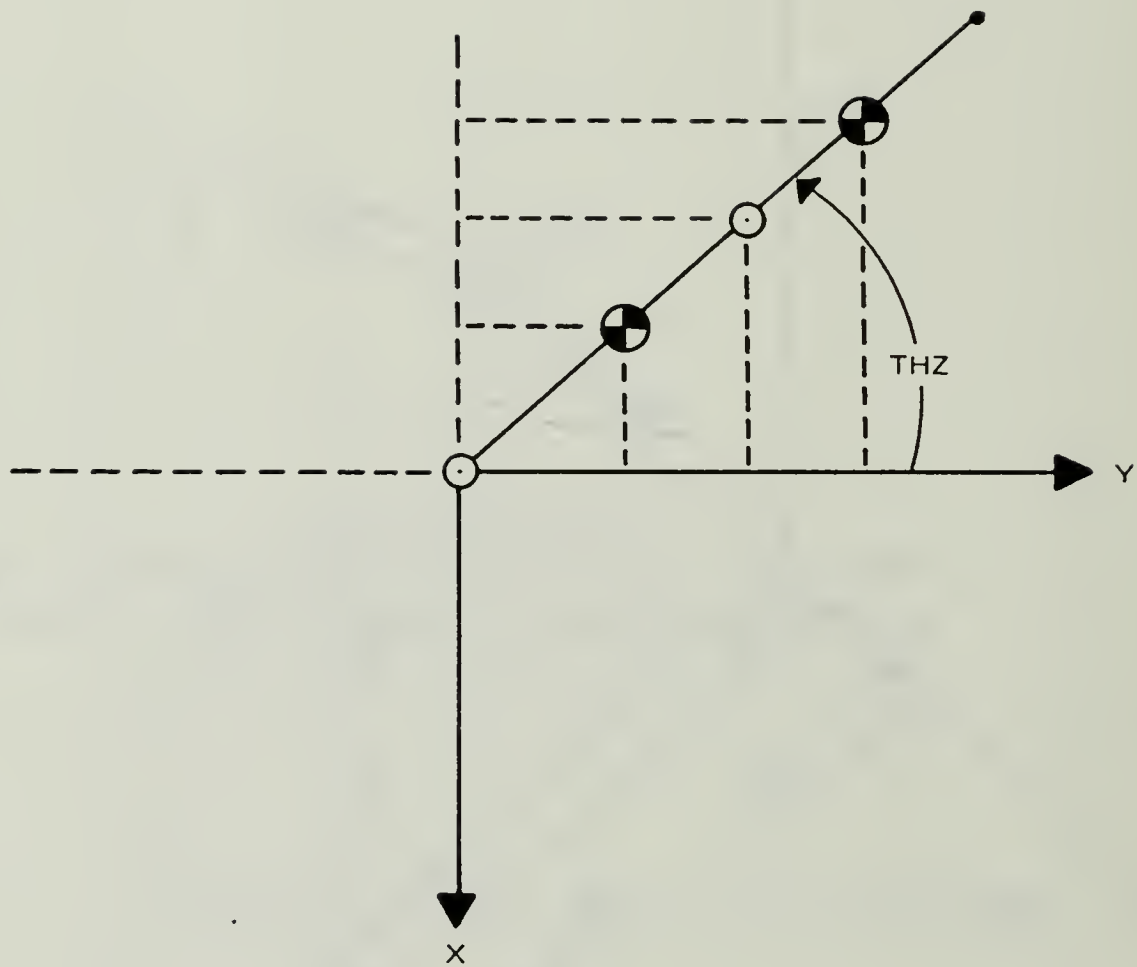


Figure 6. Gravitational Torque X and Y Components

With the given constraints, this was done in the following manner. Angle THZ was calculated from the second integral of WDZ1 (the base link only revolved around its vertical axis). Then its cosine and sine were used to obtain the X and Y components of the gravitational equilibrium torque as follows:

$$TGX = TG * \cos (THZ) \quad (43)$$

$$TGY = TG * \sin (THZ) \quad (44)$$

## E. INERTIAS

The inertias of the links of the robot were calculated the first time from the equations of Reference 2 in the global (inertial) reference frame and then transferred to the local coordinates (Figure 7) by the use of a transformation [Ref. 9], giving the following equation:

$$\text{Inertia}(\text{local}) = \text{TMAT} * \text{Inertia}(\text{global}) * \text{TMAT}^T \quad (45)$$

where MATA is a transformation matrix based on the link angles [Ref. 1 and Ref. 10]. Knowing that the local inertias remain constant, the global inertias for each link were calculated for each time iteration by the use of the inverse transformation below:

$$\text{Inertia}(\text{global}) = \text{TMAT}^T * \text{Inertia}(\text{local}) * \text{TMAT} \quad (46)$$

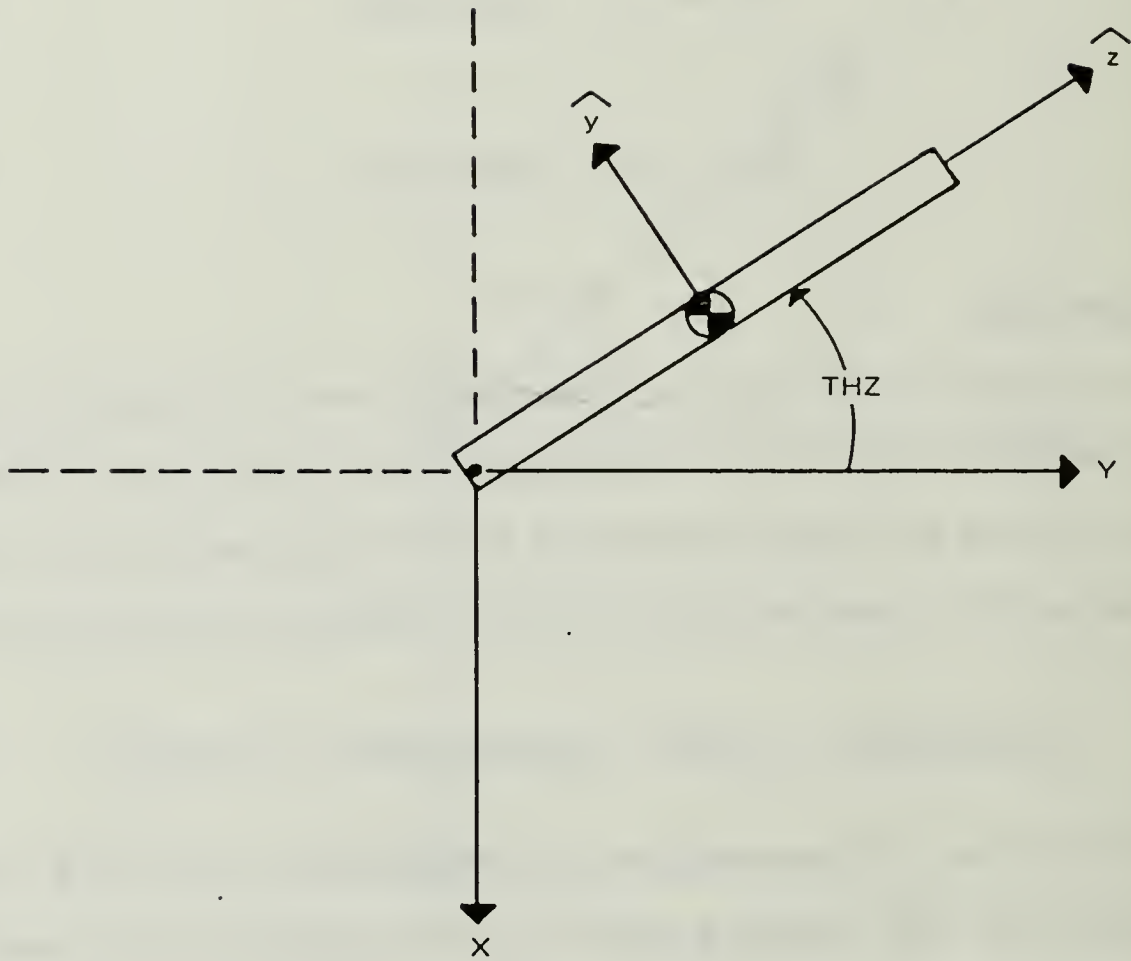


Figure 7. Inertia Coordinate Systems

## IV. SIMULATION NONSINGULAR MOTION VALIDATION

### A. TWO-DIMENSIONAL MOTION VALIDATION

A known motion which had adjacent links aligned was that of the two-dimensional double pendulum (Figure 8). A comparison of the simulation to the theoretical solution was used to validate the nonsingularity, the motion tracking, and the gravitational effects of the simulation.

The theoretical solution [Ref. 11] used a Lagrange equation which only included the linear kinetic energy of the system (equation 47), while the simulation included both the linear and rotational kinetic energies (equation 48). To account for the difference, the inertia in equation 48 was set to a small value. Note that it was not possible to set the inertias to zero because the inertia entries in matrix MATA were on the diagonal and would thus not have allowed the inverse to be calculated.

$$\text{Kinetic Energy} = 1/2 * m * v^2 \quad (47)$$

$$\text{Kinetic Energy} = 1/2 * m * v^2 + 1/2 I * w^2 \quad (48)$$

The point mass method developed in Reference 3 allowed the first mass of the double pendulum to be set to a value equal to that of

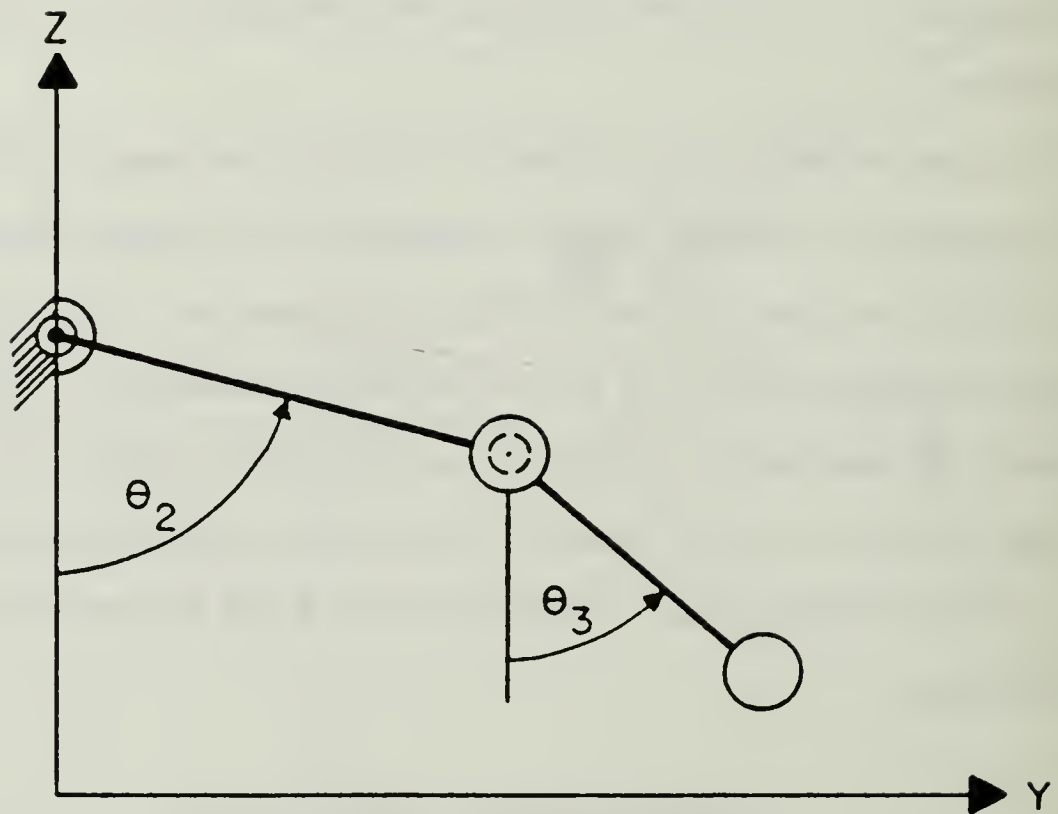


Figure 8. **Double Pendulum Configuration**



the outer point mass of link 2 plus the inner point mass of link 3 (equation 49). The second mass of the double pendulum was set to the value of the outer point mass of link 3 (equation 50).

$$m1(dp) = m(2,2) + m(3,1) \quad (49)$$

$$m2(dp) = m(3,2) \quad (50)$$

The lengths of the double pendulum were the same as the link lengths (Figure 8).

The results of the theoretical and simulation program for the double pendulum problem agreed (Figures 9 and 10), and would have been identical had the inertia been set to zero. Tracking of the motion through the potential singular points presented no difficulties to the simulation (Figure 11).

## **B. THREE-DIMENSIONAL NONSINGULAR MOTION**

A motion which had potential singular points was chosen which validated the nonsingularity of the simulation in three dimensions (Figure 12). The motion which was chosen was to input the joint 1 and joint 2 angles as a time function and allow the third joint angle to remain zero, so links 2 and 3 were always aligned and potentially singular. This chosen motion also had all three links aligned at time equal to 0.785 seconds (Figure 12).

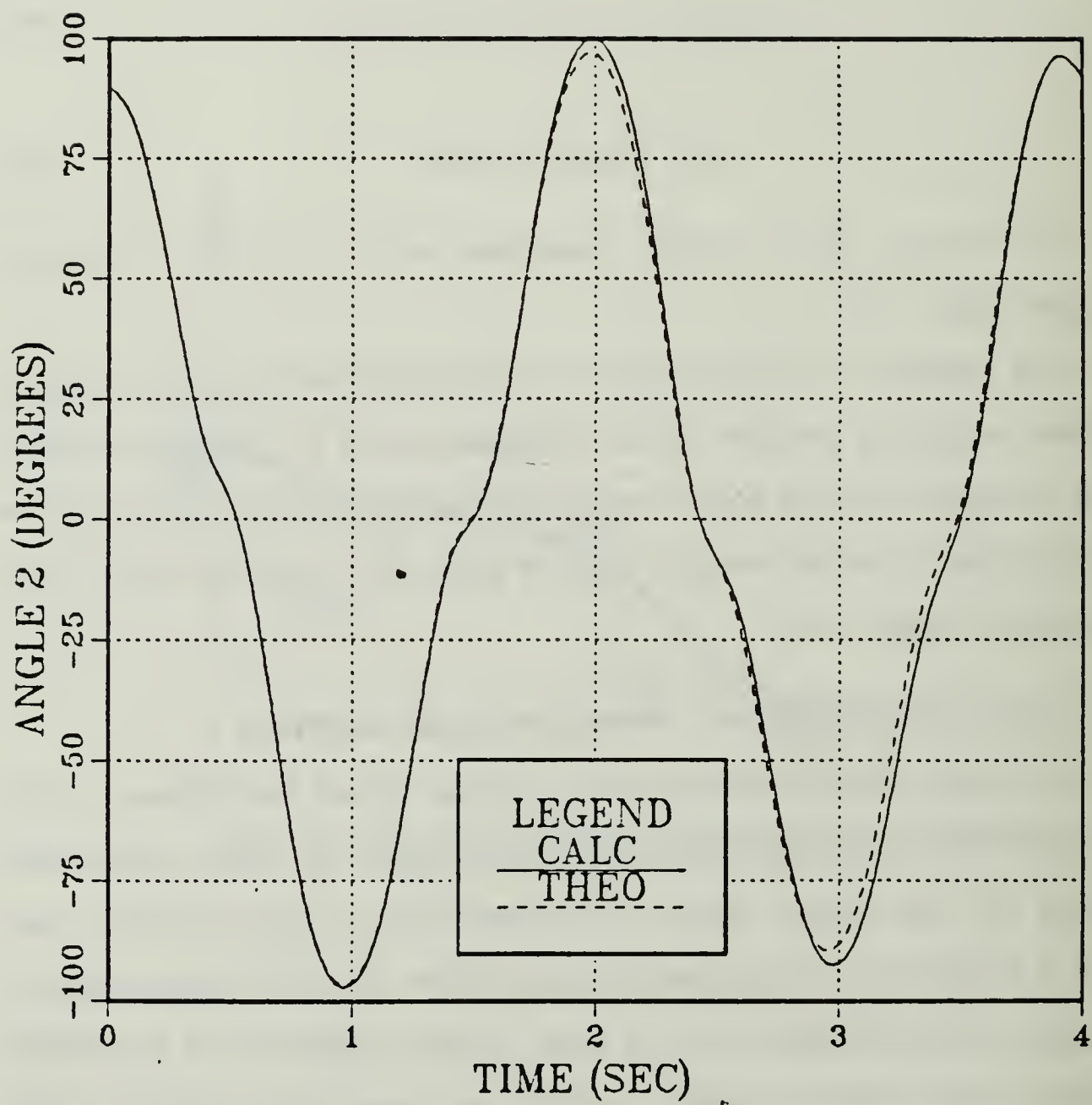


Figure 9. Double Pendulum Angle 2

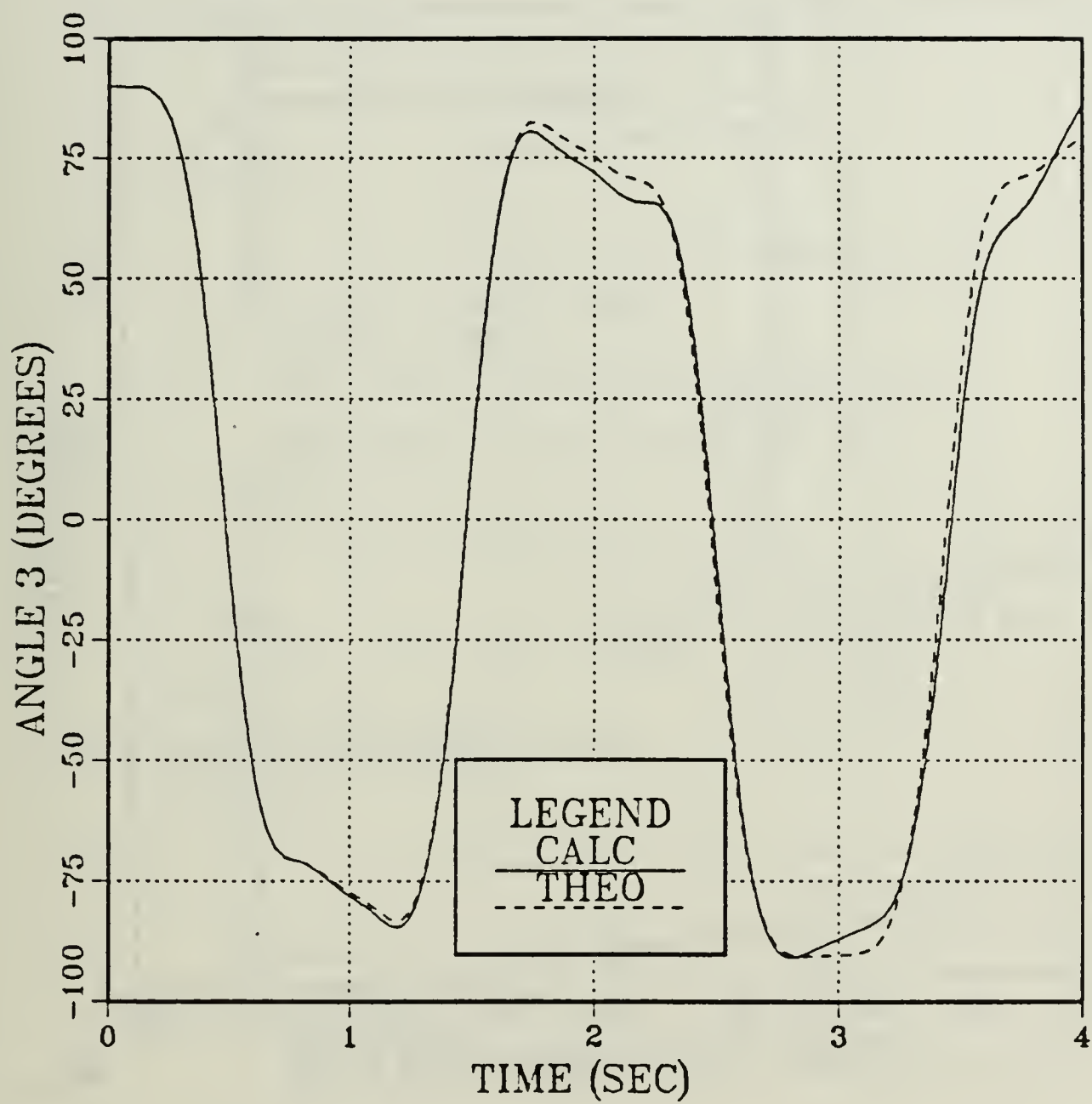
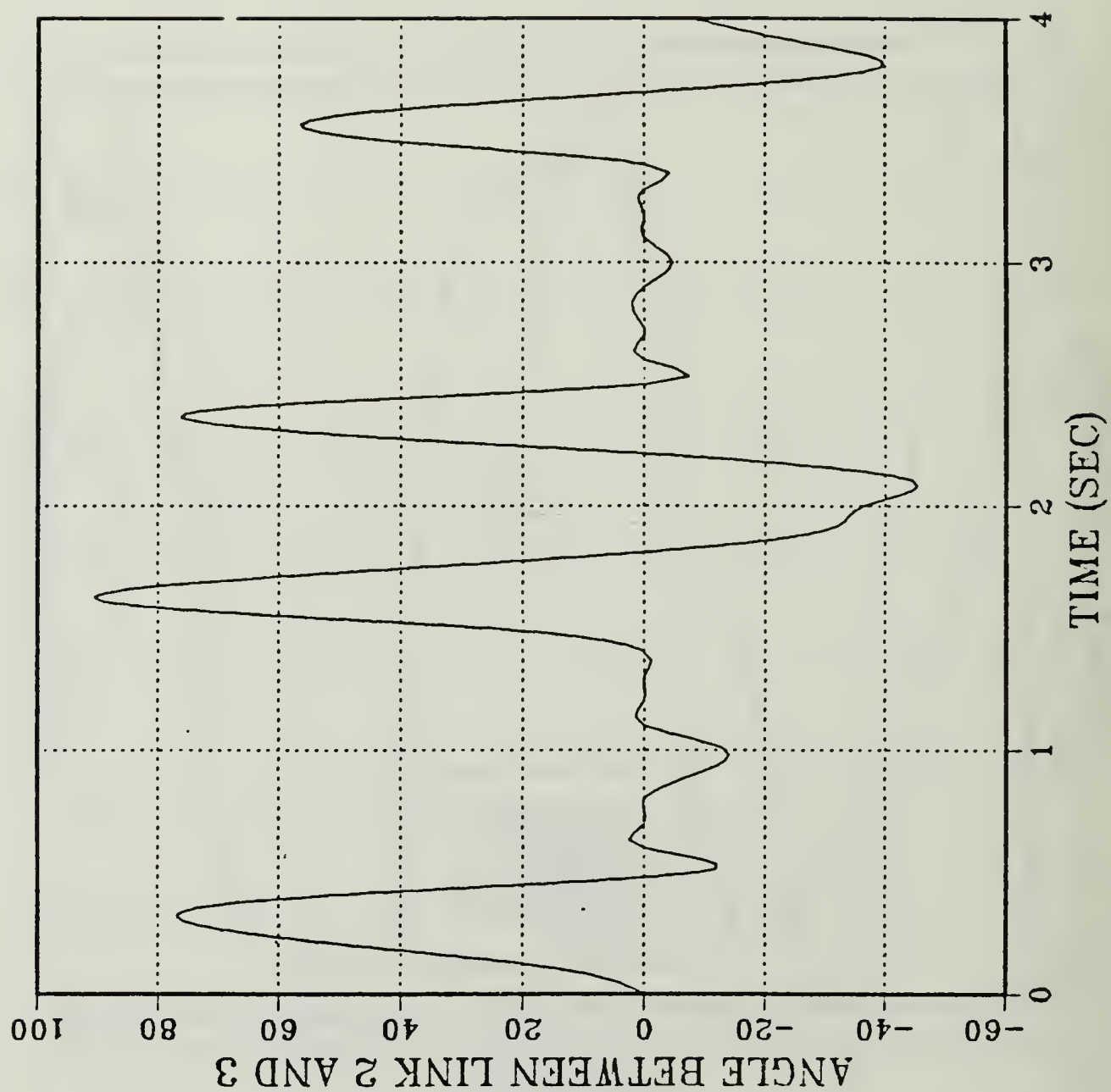


Figure 10. Double Pendulum Angle 3



**Figure 11. Double Pendulum Simulation Through  
Potential Singular Points (angle = 0)**

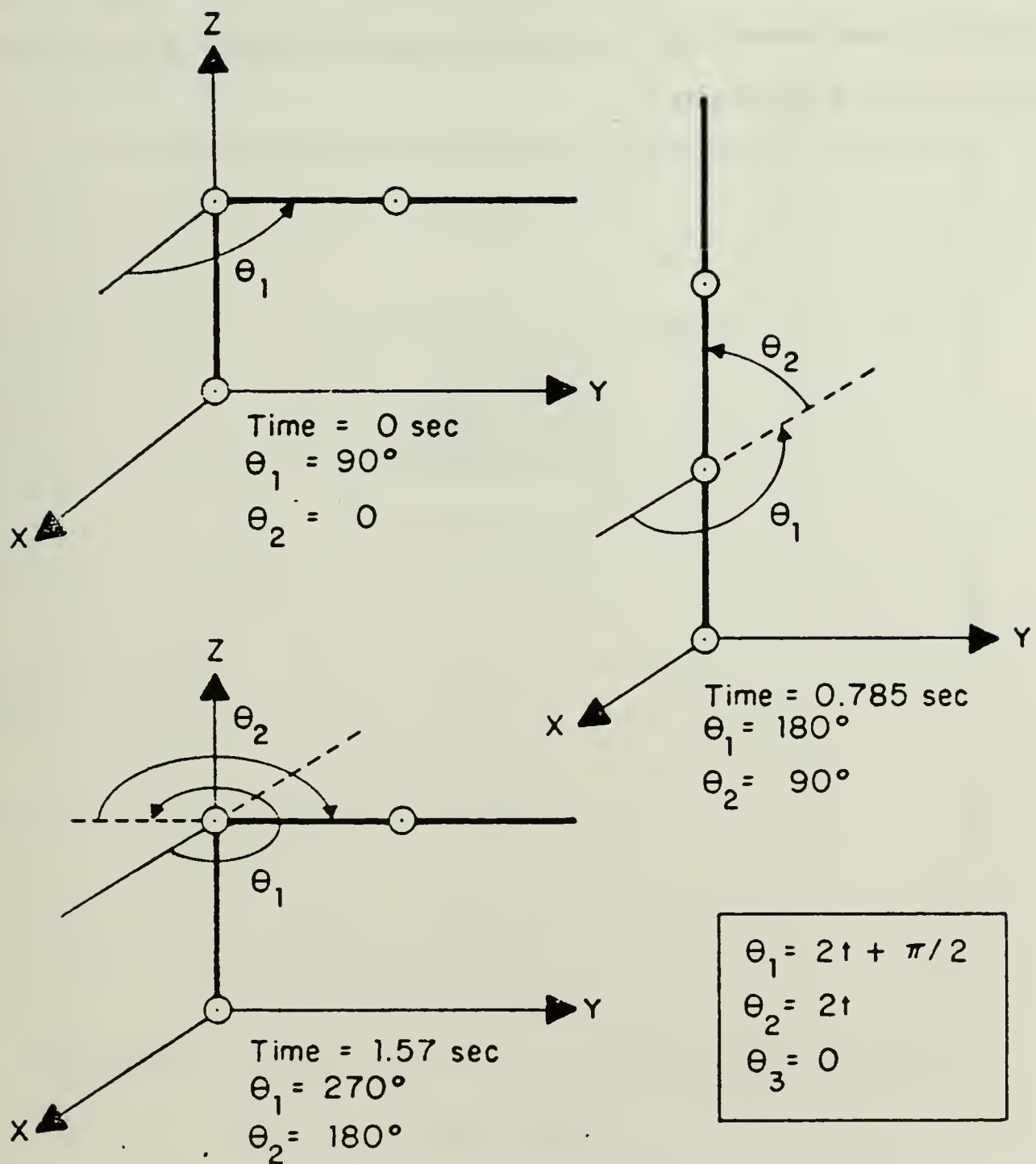


Figure 12. Three-Dimensional Nonsingular Validation



The simulation again had no difficulties tracking the input motion through the potentially singular points (Figure 13). The error in the predicted and simulated end effector position (Figure 14) was never greater than 1.0 percent.

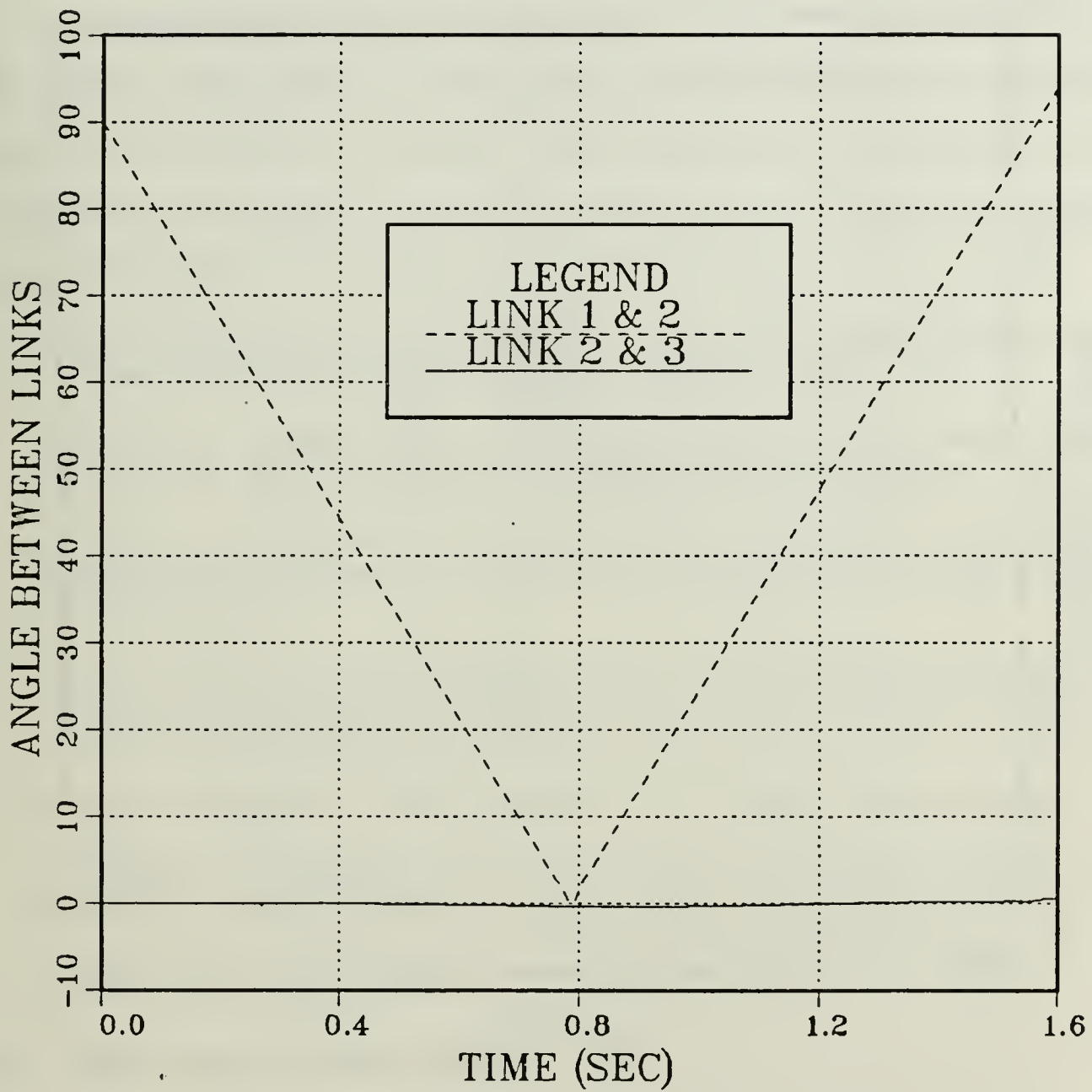


Figure 13. Three-Dimensional Potential Singular Points

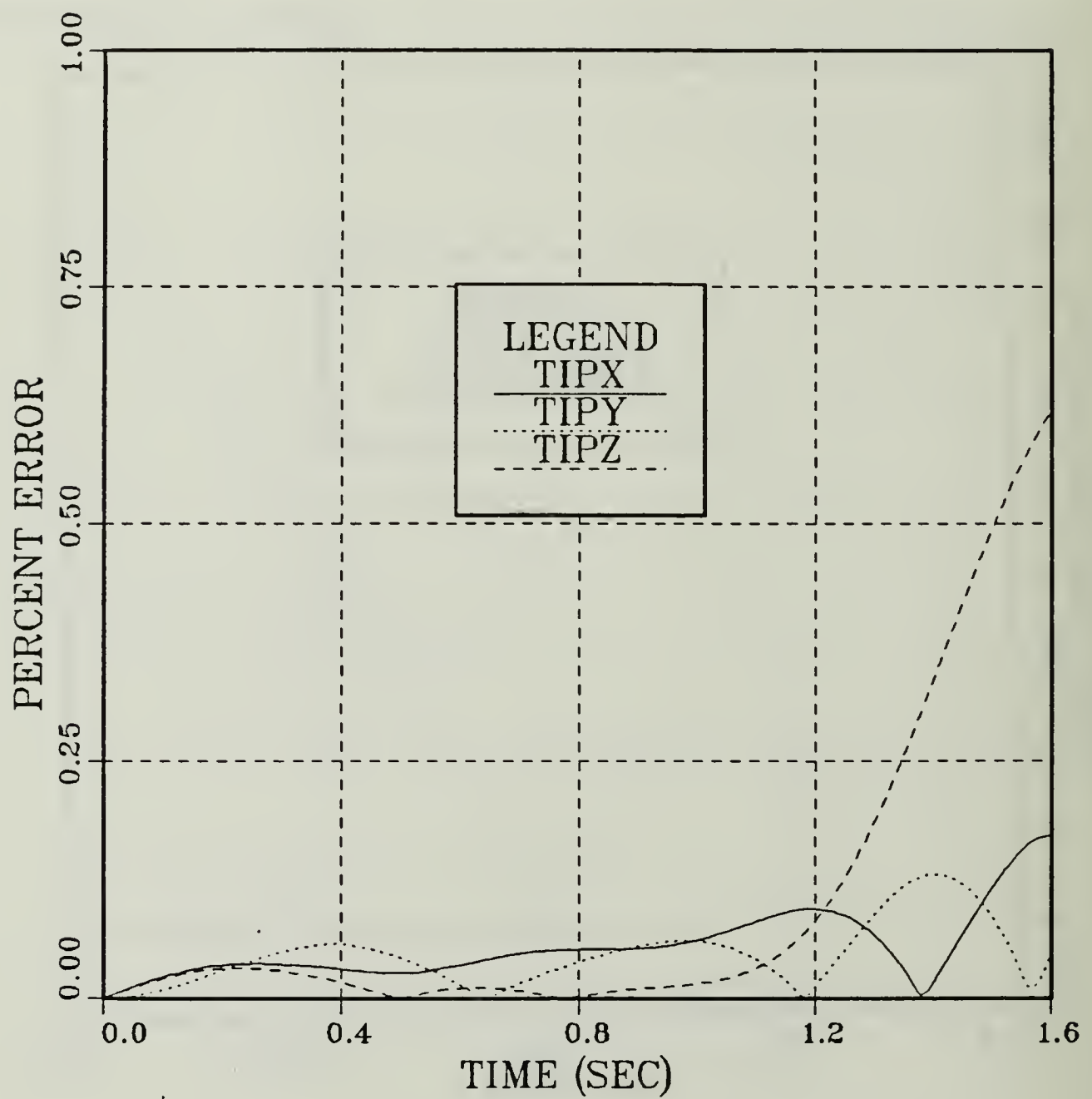


Figure 14. End Effector Position Error

## **V. EXPERIMENTAL VALIDATION**

### **A. MANIPULATOR**

A robot manipulator was chosen to check the agreement of the simulation to data from an actual robot arm. The arm that was available was the NEPTUNE II (Figure 15), which is a low-performance, hydraulic-operated arm. This arm required certain hardware modifications, namely:

- the remounting of the electronic solenoid boards to facilitate access and provide protection from hydraulic leaks;
- installation of bleed holes in the major joint pistons to allow air to be bled off when the system has been opened for repairs;
- design and manufacture of a pump and accumulator stand that would provide spray protection for the manipulator in the case of malfunctions;
- modification to the plumbing of the pump and accumulator to include an electrically controlled solenoid valve operated from the Electronic Control Panel (ECP);
- rewiring the pump to be electrically controlled from the ECP;
- redesign and manufacture of the waist joint (joint 0) shaft and coupling, along with that of several piston end plates; and
- an overhaul of the complete electronics.

### **B. PRESSURE TRANSDUCERS**

Pressure transducers which had previously been installed on the NEPTUNE II arm at joints 1 and 2 (Figure 16) allowed for data collection to validate the two-dimensional simulation with gravitational effects. Initially, the pressure transducers had to be calibrated.



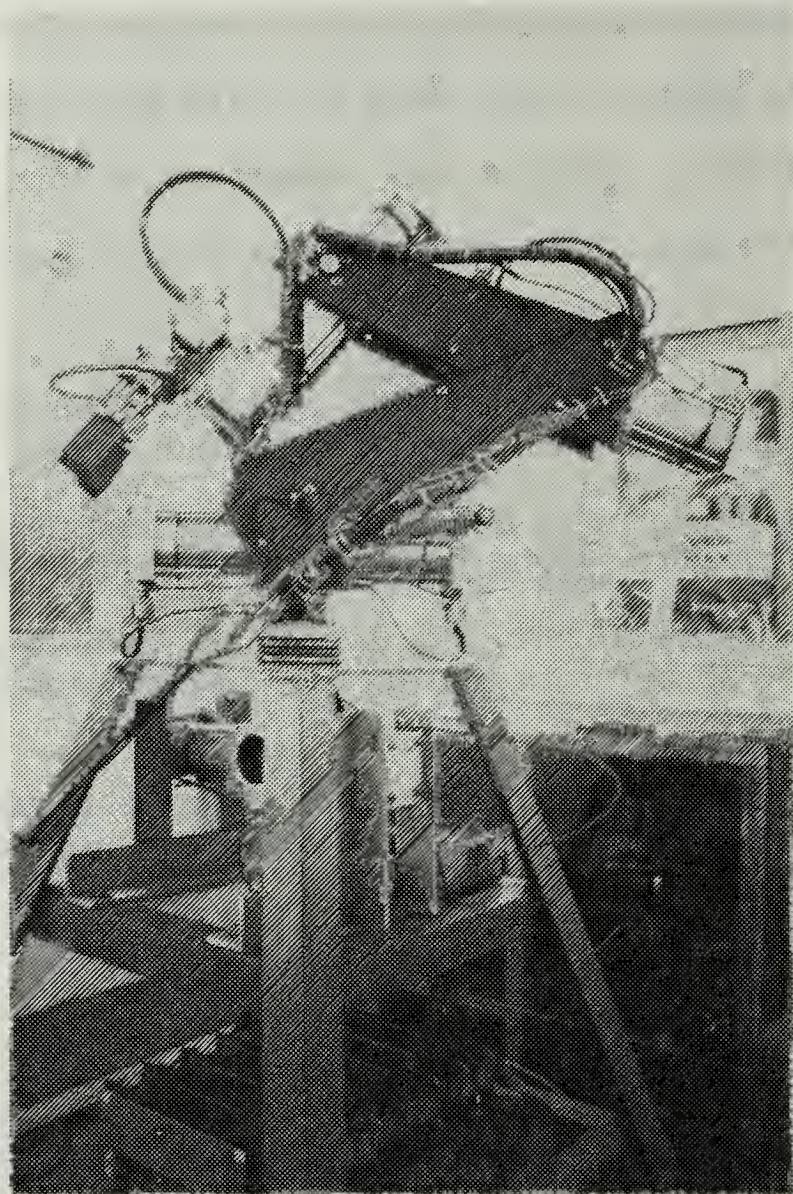
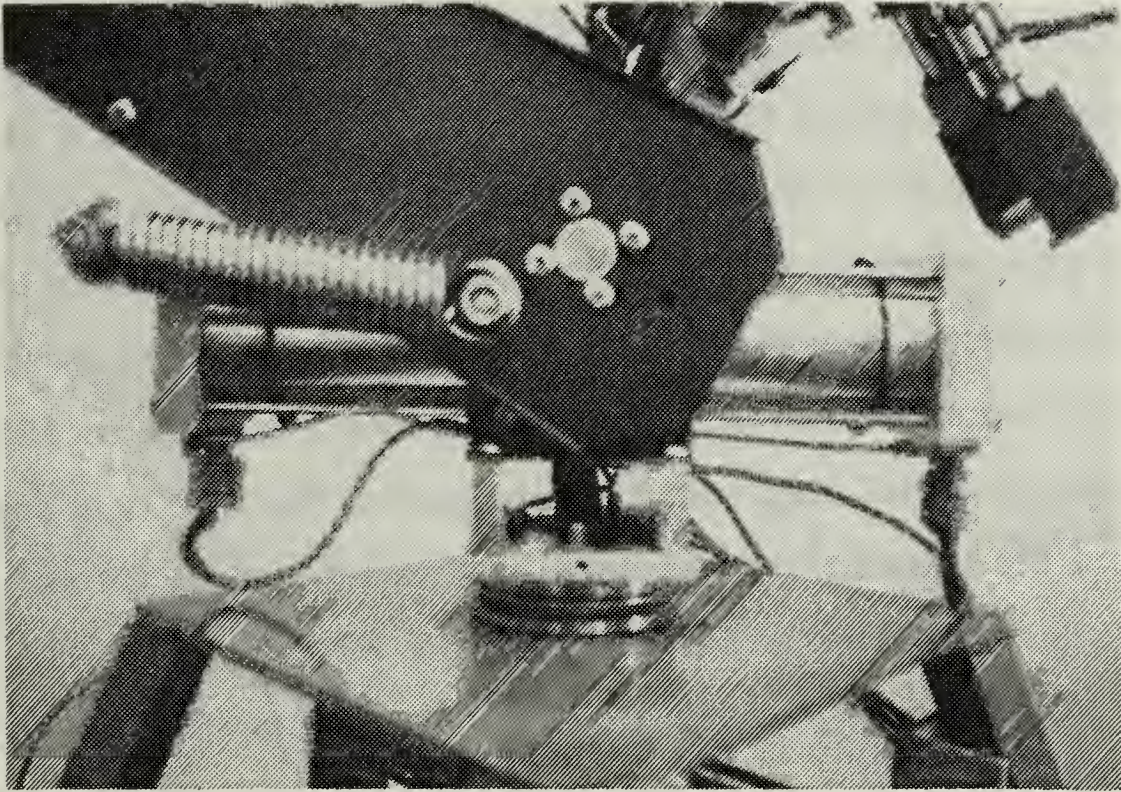
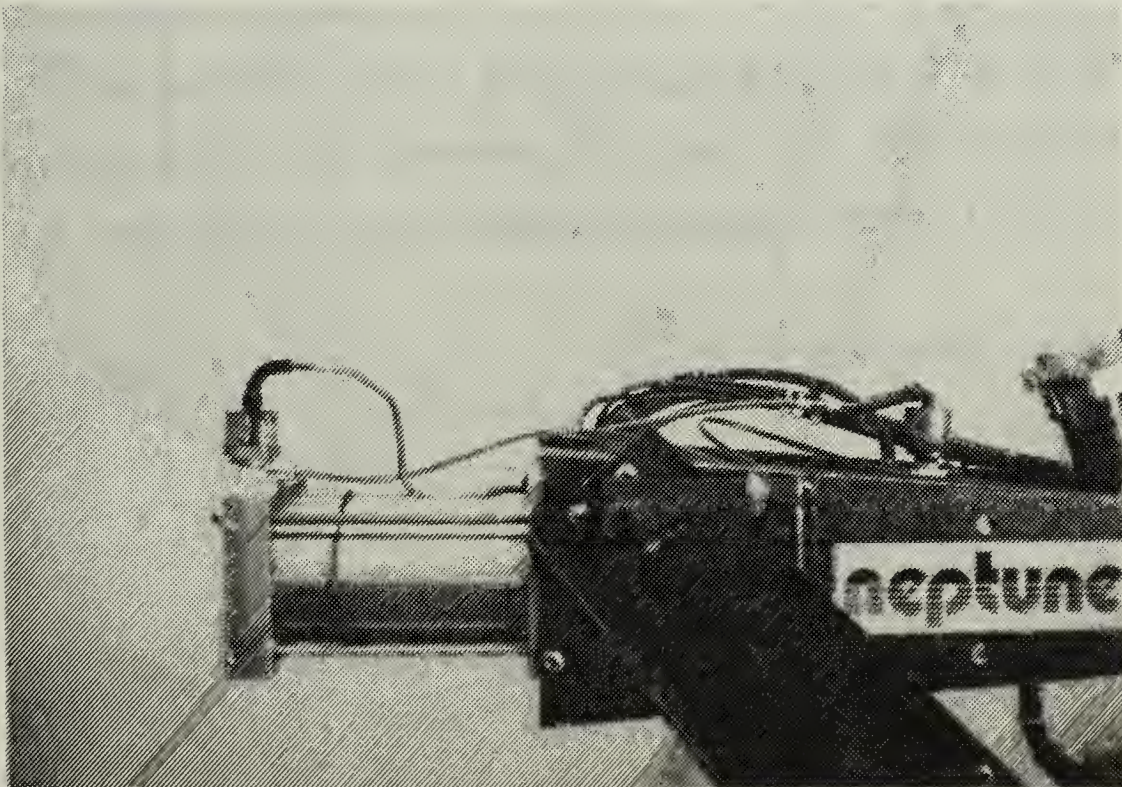


Figure 15. **NEPTUNE II Robot**





Joint 1



Joint 2

Figure 16. **Robot Pressure Transducers**



A 3,000-pound nitrogen bottle with a 200 psi regulator was used for pressure transducer calibration. This was connected to a test rig which consisted of a varlinear pressure adjustment and a 200-pound pressure gage which had been calibrated and was incremented in 0.2-pound increments (Figure 17). The varlinear allowed the pressure at the transducer to be varied in fine increments from zero to 120 psi, which was the stated range of the transducers. The electrical connections from the transducers had been hard-wired to the ECP and then to a digital multimeter (Fluke 8024 B); the meter 20-volt scale was used to read the voltage output of the transducers (the voltage was read to the second decimal place). Each Omega Series PX302 pressure transducer had two complete sets of data from 0 to 120 psi; the averages of these are provided in Table 1. The pressure transducers were very linear from 0 to 115 psi, with each 5 psi producing a voltage of 0.5 volts (Figure 18). This range of pressures was well acceptable for the NEPTUNE II, which operates between 0 and 118 psi [Ref. 12].

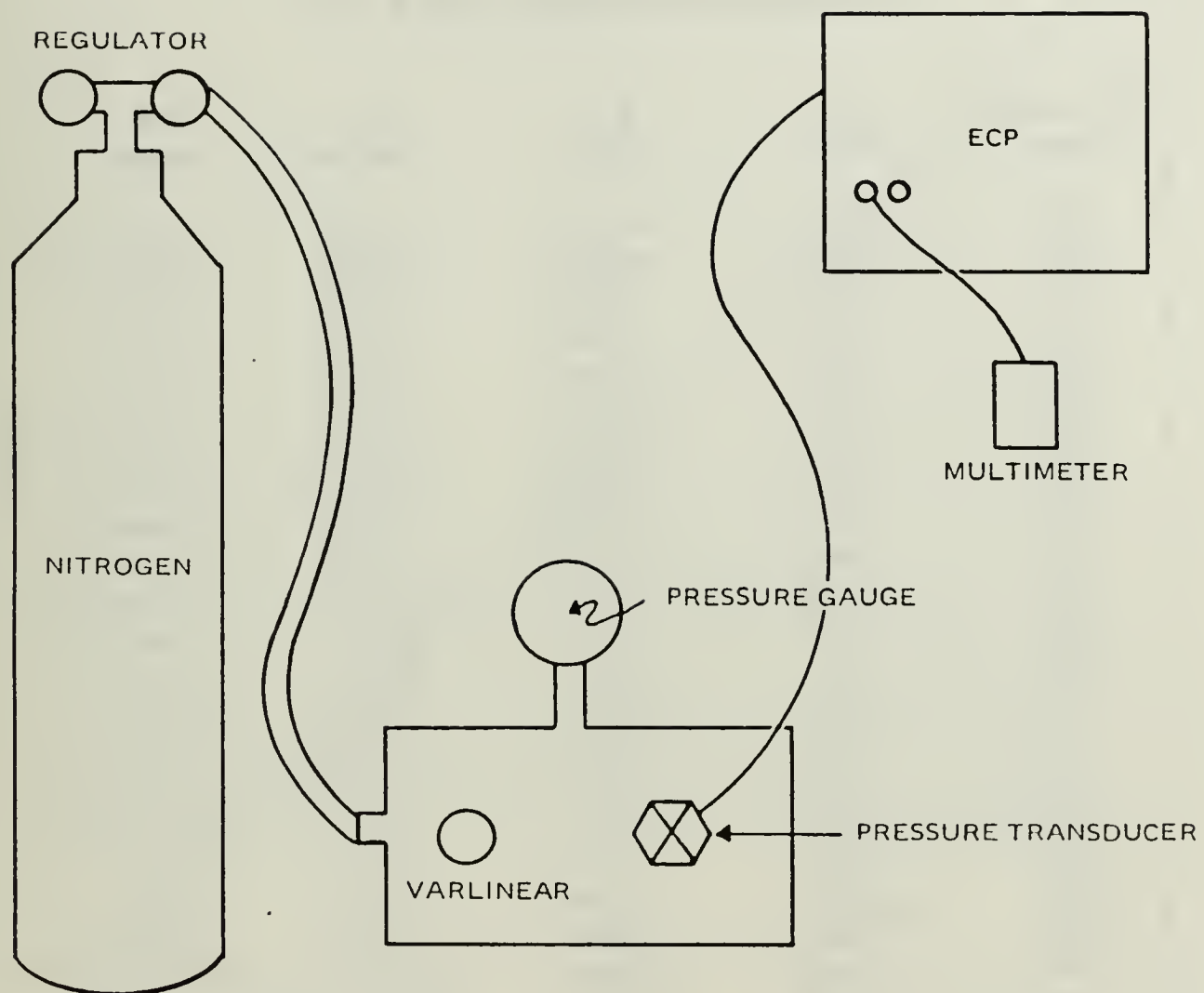


Figure 17. Pressure Transducer Calibration Set-Up

TABLE 1  
PRESSURE TRANSDUCER DATA

Pressure (psi)	P1 volts	P2 volts	P3 volts	P4 volts
000	-0.010	0.000	0.015	0.000
005	0.500	0.510	0.520	0.500
010	0.995	1.005	1.025	1.005
015	1.500	1.500	1.530	1.500
020	1.985	1.995	2.020	2.000
025	2.500	2.500	2.530	2.500
030	3.000	3.000	3.030	3.000
035	3.490	3.500	3.530	3.490
040	3.990	3.995	4.030	3.995
045	4.500	4.500	4.530	4.500
050	4.995	4.995	5.025	4.990
055	5.490	5.500	5.520	5.490
060	5.995	5.995	6.020	5.990
065	6.490	6.500	6.520	6.490
070	6.990	7.000	7.020	6.990
075	7.490	7.500	7.510	7.500
080	7.990	8.005	8.010	8.000
085	8.490	8.510	8.510	8.500
090	8.990	9.005	9.000	9.000
095	9.480	9.500	9.500	9.500
100	9.990	10.005	10.000	10.000
105	10.490	10.510	10.500	10.500
110	10.990	11.010	11.000	11.000
115	11.485	11.490	11.490	11.500
120	11.580	11.490	11.990	12.000

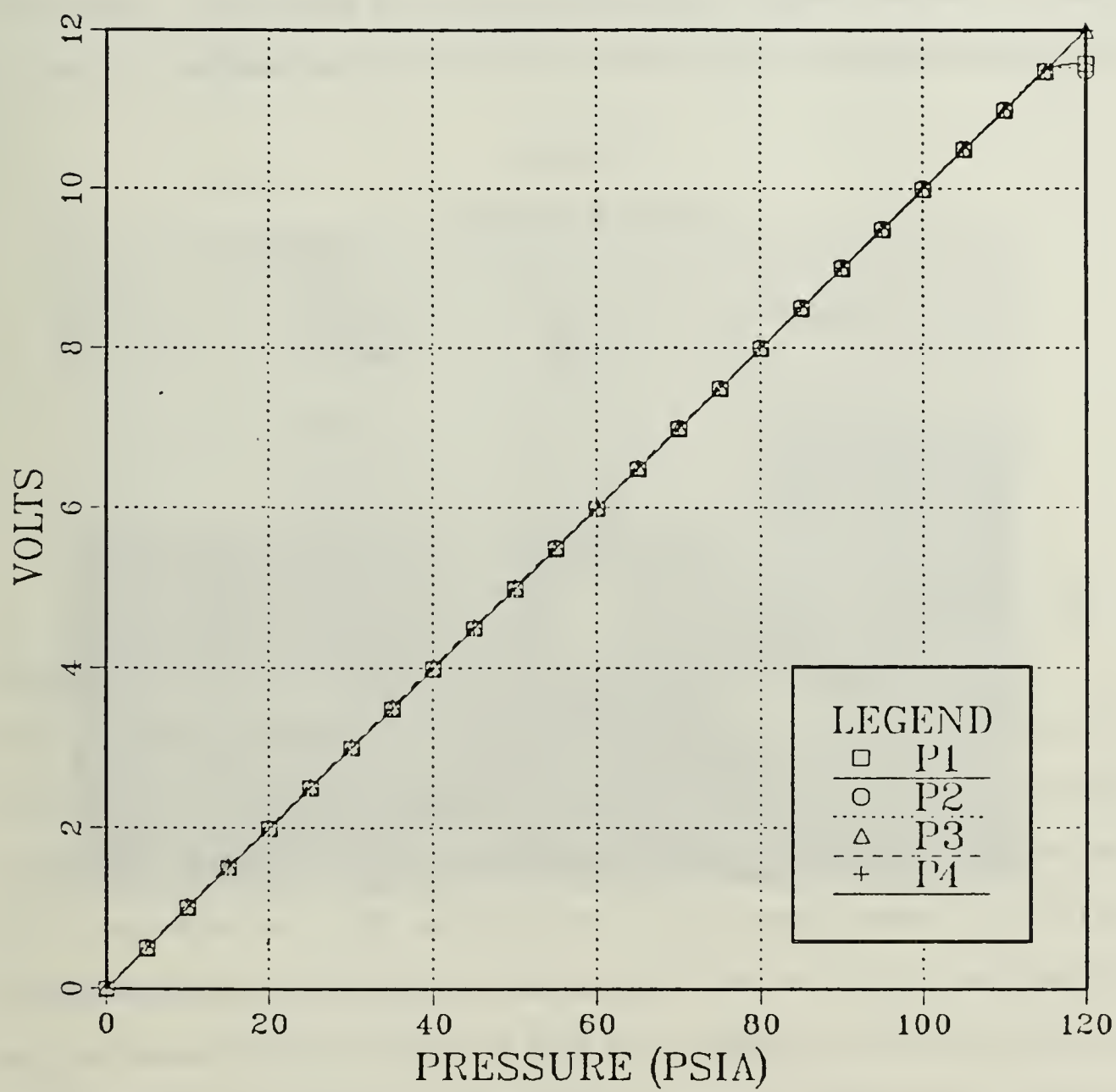


Figure 18. Pressure Transducer Data



## C. MEASUREMENTS

### 1. Link Lengths

With the necessity to disassemble the NEPTUNE II for repairs, the link length measurements were physically measured. These measurements are provided in Table 2 to the nearest 1/8 inch.

TABLE 2  
LINK LENGTHS

Parameter	Length inch	Length meter
LINKL1	10.250	0.2604
LINKL2	16.375	0.4159
LINKL3	13.750	0.3498

### 2. Link Masses

Also during the disassembly of the robot, the link masses were measured using a scale which was in half-pound increments. The design of the robot allowed for a good approximation of the two masses per link as used in the simulation (Figures 19 and 20). This was a crude measurement, but it was felt that this was sufficient to be used for the simulation and that the time necessary to completely disassemble the robot in order to be able to obtain closer measurements was not justified. The masses which were measured are provided in Table 3.

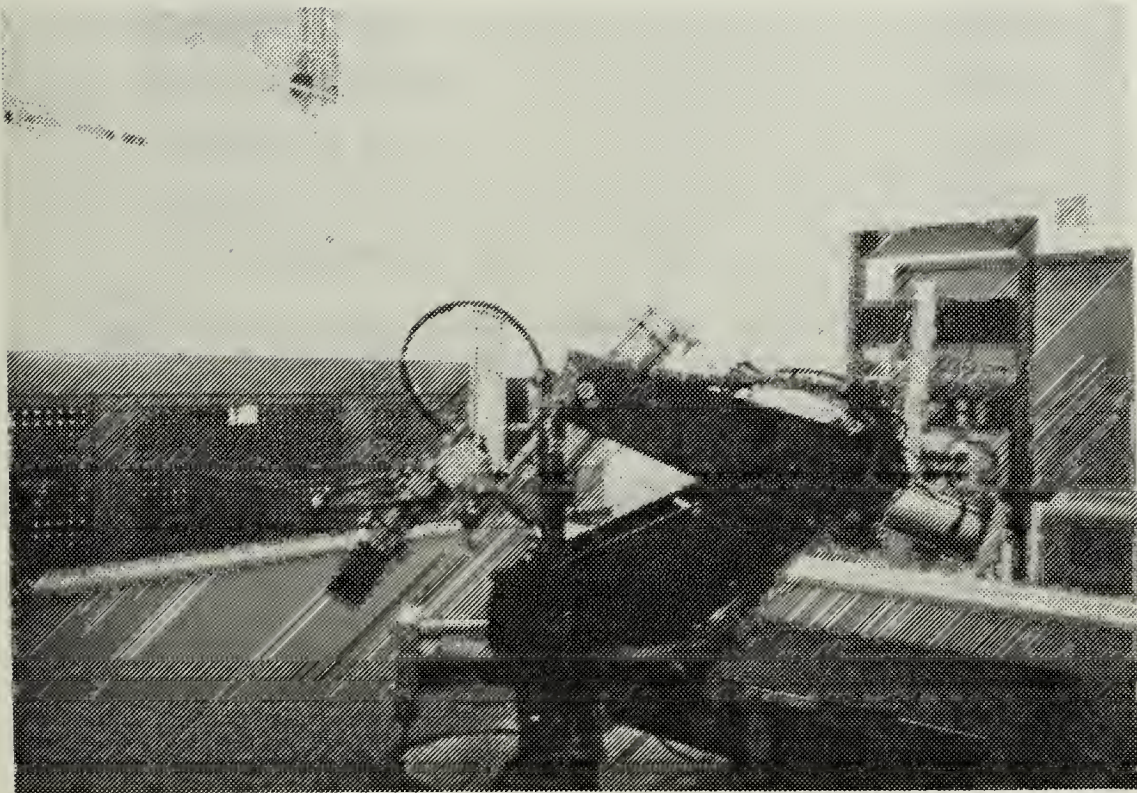


Figure 19. **Robot Link Masses**

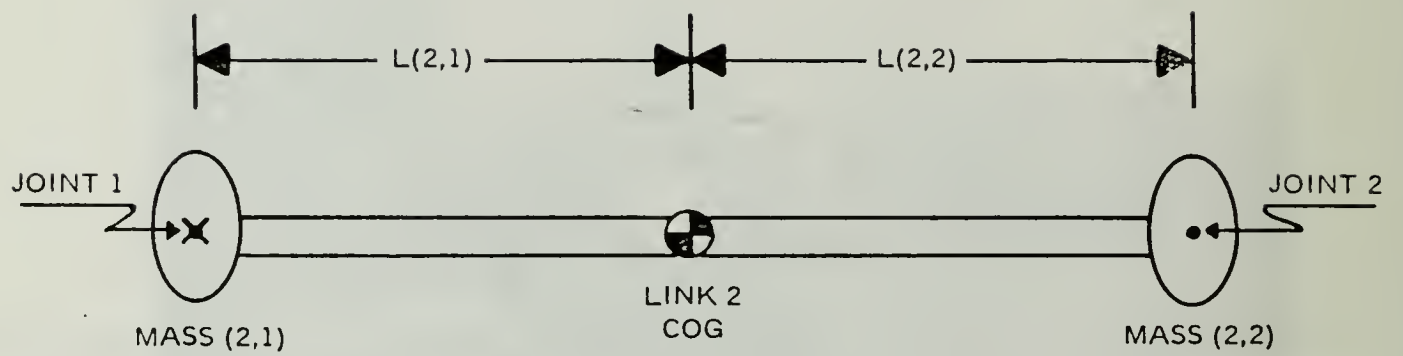


Figure 20. Simulation Link Masses

TABLE 3  
LINK MASSES

Parameter	Pounds	Kilograms
MASS(1,1) est.	5	2.273
MASS(1,2) est.	15	6.818
M1 (measured)	20	9.091
MASS(2,1) est.	1	0.455
MASS(2,2) est.	1	0.455
M2 (measured)	2	0.910
MASS(3,1) est.	13	5.909
MASS(3,2) est.	13	5.909
M3 (measured)	26	11.818

### 3. Length from COG of Link to Center of Mass

The point mass centers were assumed to be at the joints (Figure 20). A simple calculation of the moments around the center of gravity was performed; this provided the data in Table 4.

TABLE 4  
LENGTH FROM LINK CENTER OF GRAVITY  
TO MASS CENTER OF GRAVITY

Parameter	Length inch	Length meter
L(1,1)	7.6875	0.1953
L(1,2)	2.5625	0.0651
L(2,1)	8.1875	0.2080
L(2,2)	8.1875	0.2080
L(3,1)	6.8750	0.1746
L(3,2)	6.8750	0.1746



#### **D. DATA COLLECTION**

Position data and pressure data were taken for several movements of links 2 and 3 of the robot from the electronic control panel and recorded on a four-channel strip chart recorder. An example of the recorded data is provided (Figure 21).

#### **E. DATA REDUCTION**

The data taken for the various runs was reduced using the following equation to obtain joint torques.

$$\text{TORQUE} = \Delta \text{ PRESSURE} * \text{AREA}(\text{piston}) * \text{RADIUS}(\text{lever arm}) \quad (51)$$

where  $\text{AREA}(\text{piston}) = 0.00312$  square meters and  $\text{RADIUS}(\text{lever arm}) = 0.0597/2$  meters.

#### **F. VALIDATION**

The delta pressure data versus time was entered into the robot validation program as a time function and the torque was calculated as a function of time from this data. The simulation position results agreed with the actual robot position in the general trend of motion (Figure 22). It was felt that the inaccuracies in the results were due to both the simulation round-off errors and the crude methods used in the collection of the robot link masses and link lengths, particularly the lengths from the center of gravity of the link to the center of gravity of the point masses. The scalloped shape of the simulated result was attributed to the normalization of the directional cosines.

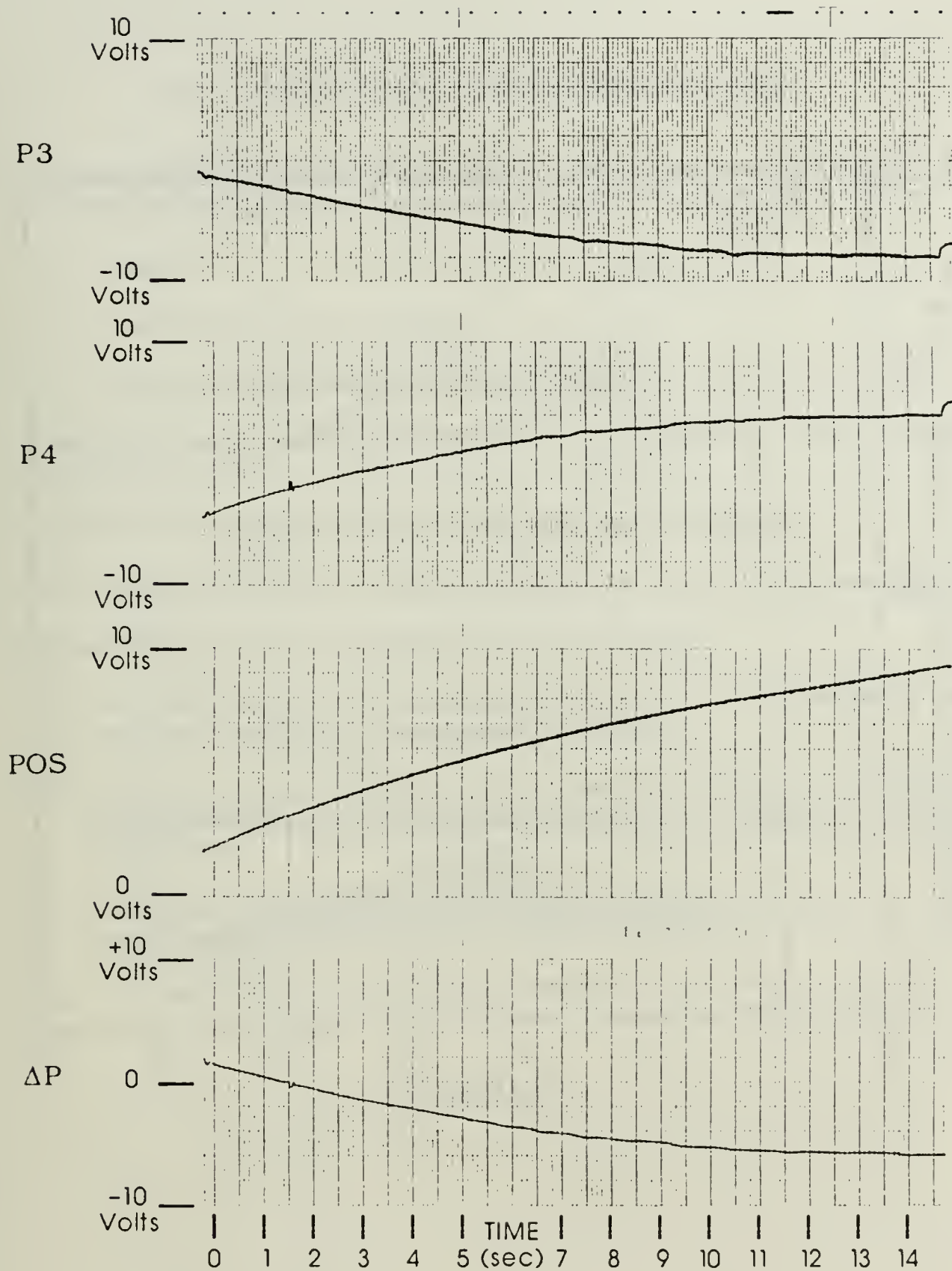


Figure 21. Pressure Recordings



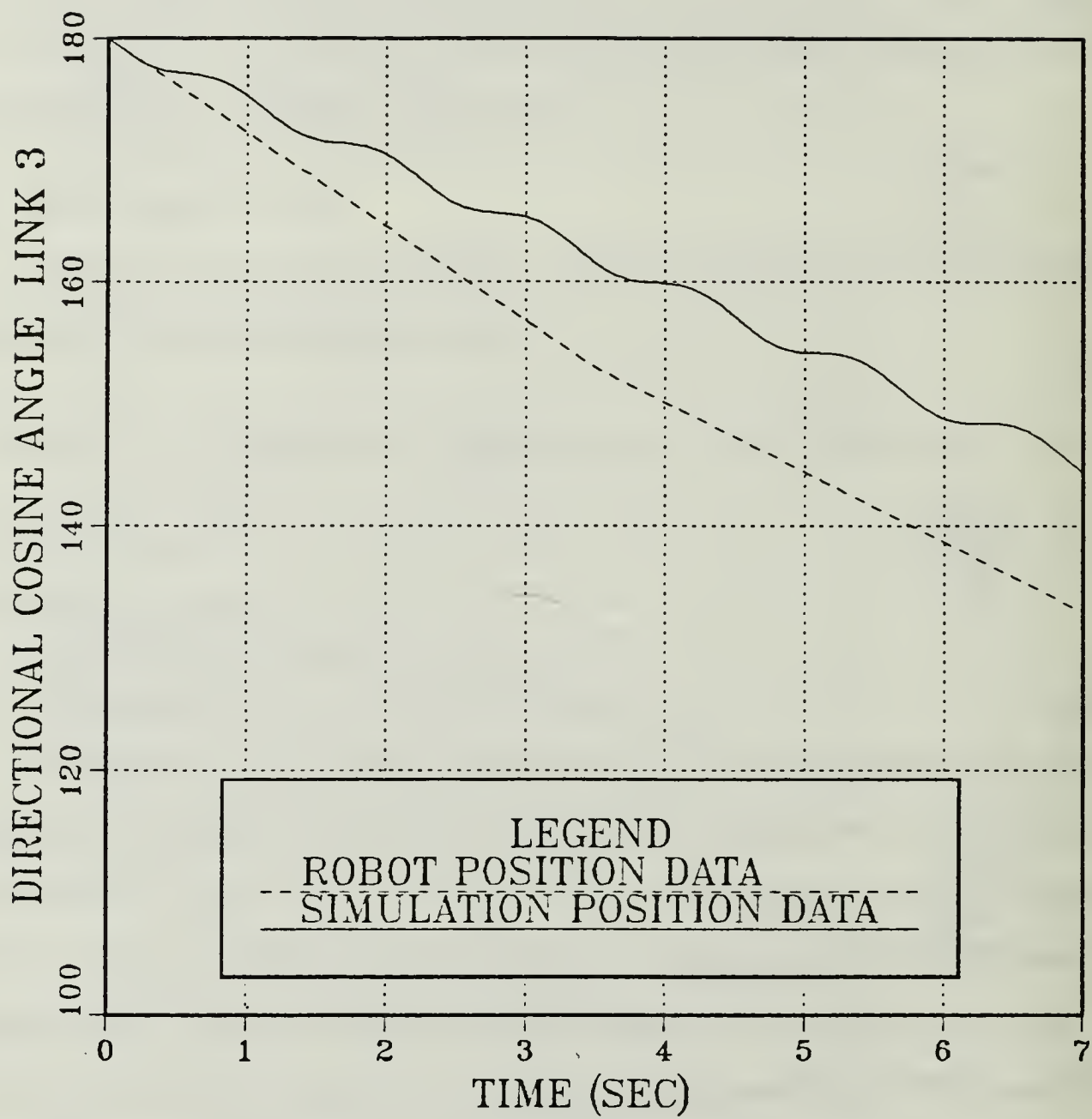


Figure 22. Robot/Simulation Motion

## VI. RESULTS AND RECOMMENDATIONS

1. A nonsingular dynamic simulation of a rigid revolute three-link manipulator has been further developed. It includes:
  - Gravity,
  - Three-Dimensional Motion,
  - Comparatively straightforward dynamics.
2. Accuracy and nonsingularity has been validated with a double pendulum.
3. Three-dimensional nonsingularity has been validated.
4. Procedures for actual robot data collection have been developed.
5. The following recommendations are provided:
  - Simulate actual three-dimensional robot performance and compare predicted to measured variables.
  - Enhance the computer program code to enable it to be more interactive. That is, let the user specify the constraints during execution.
  - Use this simulation to develop advanced controllers for rigid robot linkages.

## APPENDIX A

### DIRECT DYNAMIC SIMULATION PROGRAM

TITLE DIRECT DYNAMICS PROBLEM

```
*****
* THIS IS A PROGRAM THAT WILL SOLVE THE DIRECT DYNAMICS PROBLEM FOR A *
* 3 LINK RIGID BODY MANIPULATOR WITHOUT THE USE OF ANY TRANSFORMATION *
* MATRICES. THE ORIGINAL PROGRAM WAS WRITTEN BY LT ATINOK AND HAS *
* HAS BEEN GREATLY MODIFIED TO INCLUDE 3 D MOTION AND GRAVITATIONAL *
* EFFECTS BY LT R. M. VERBOS USN SEPTEMBER 1988 *
*****
```

TERMINAL

METHOD RKSFX

```
PRINT .10,DRCANX(1),DRCANY(1),DRCANZ(1),...
DRCANX(2),DRCANY(2),DRCANZ(2),DRCANX(3),DRCANY(3),DRCANZ(3),...
DRCSX(1),DRCSY(1),DRCSZ(1),DRCSX(2),DRCSY(2),DRCSZ(2),DRCSX(3),...
DRCSY(3),DRCSZ(3),...
JX0,JY0,JZ0,JX1,JY1,JZ1,JX2,JY2,JZ2,...
LCOGX(1),LCOGY(1),LCOGZ(1),LCOGX(2),LCOGY(2),LCOGZ(2),...
LCOGX(3),LCOGY(3),LCOGZ(3),...
TOX,TOY,TOZ,T1X,T1Y,T1Z,T2X,T2Y,T2Z,...
WDX(1),WDY(1),WDZ(1),WDX(2),WDY(2),WDZ(2),WDX(3),WDY(3),WDZ(3),...
AX1,AY1,AZ1,AX2,AY2,AZ2,AX3,AY3,AZ3,...
W1X,W1Y,W1Z,W2X,W2Y,W2Z,W3X,W3Y,W3Z,...
FX0,FY0,FZ0,FX1,FY1,FZ1,FX2,FY2,FZ2,...
IXXT(1),IYYT(1),IZZT(1),IXXT(2),IYYT(2),IZZT(2),IXXT(3),IYYT(3),...
IZZT(3),IXYT(1),IYZT(1),IXZT(1),IXYT(2),IYZT(2),IXZT(2),IXYT(3),...
IYZT(3),IXZT(3),...
LNCOG1,LNCOG2,LNCOG3,THZANG,COSTHZ,SINTHZ,TIPX,TIPY,TIPZ,...
ANG12X,ANG12Y,ANG12Z,ANG23X,ANG23Y,ANG23Z,...
T1CH,T2CH,IER
CONTROL FINTIM =1.5, DELT = .0005, DELMAX = .1, DELPRT = .10
D DIMENSION MATA(27,27),MASS(3,2),L(3,2),RX(3,2),RY(3,2),RZ(3,2)
D DIMENSION IXX(3,2),IXZ(3,2),IXY(3,2),IYY(3,2),IYZ(3,2),IZZ(3,2)
D DIMENSION IMAT(3,3,3),NIMAT(3,3),TMAT(3,3),TMATTR(3,3),MATTMP(3,3)
D DIMENSION LIMAT(3,3,3)
FIXED IER,I,J,M,K,P,N,IA,IDGT,A,I1
ARRAY MATB(27),ABMATB(27),LCOGX(3),LCOGY(3),LCOGZ(3)
ARRAY VECTA0(3),VECTB0(3),VECTA1(3),VECTB1(3),VECTA2(3),VECTB2(3)
ARRAY VECTA(3),VECTB(3)
ARRAY WDX(3),WDY(3),WDZ(3),W1(3),W2(3),W3(3)
ARRAY RBG1(3),RAG1(3),RBG2(3),RAG2(3),RBG3(3)
ARRAY WKAREA(850)
ARRAY IXXT(3),IYYT(3),IZZT(3),IXYT(3),IXZT(3),IYZT(3)
```

ARRAY DRCANX(3),DRCANY(3),DRCANZ(3)

ARRAY DRCSX(3),DRCSY(3),DRCSZ(3)

INITIAL

\*\*\*\*\* INPUT \*\*\*\*\*

\* INPUT PARAMETER CONSTANTS

55       A = 15.0D0  
         P = 0.0D0  
         W = 2 \* PI  
         IDGT = 6  
         G = 9.81D0  
         N = 27  
         M = 1  
         IA = 27  
         IER = 0  
         LEVELQ = 0  
         LEVLDQ = 0

\* INPUT JOINT LOCATIONS IN METERS

JX0 = 0.0D0  
JY0 = 0.0D0  
JZ0 = 0.0D0  
JX1 = 0.0D0  
JY1 = 0.0D0  
JZ1 = 0.2D0  
JX2 = 0.0D0  
JY2 = 0.416D0  
JZ2 = 0.2D0

\* INPUT TORQUE CONSTANTS

TOX       = 0.0D0  
TOY       = 0.0D0  
TOZ       = 0.0D0  
T1X       = 0.0D0  
T1Y       = 0.0D0  
T1Z       = 0.0D0  
T2X       = 0.0D0  
T2Y       = 0.0D0  
T2Z       = 0.0D0  
TG1       = 0.0D0  
TG2       = 0.0D0  
T1FNC      = 0.0D0  
T2FNC      = 0.0D0

\* INPUT DISTANCE FROM CENTER OF LINK TO CENTER OF MASS  
\* FOR EACH LINK ENDS

L(1,1) = 0.05D0  
L(1,2) = 0.15D0  
L(2,1) = 0.20D0  
L(2,2) = 0.216D0  
L(3,1) = 0.225D0  
L(3,2) = 0.226D0

\* INPUT THE LINK LENGTHS OF THE ROBOT

LINKL1 = 0.20D0

LINKL2 = 0.416D0  
LINKL3 = 0.451D0

\* INPUT MASS AT LINK ENDS IN KILOGRAMS

110 MASS(1,1) = 11.0D0  
MASS(1,2) = 33.0D0  
MASS(2,1) = 2.2D0  
MASS(2,2) = 2.2D0  
MASS(3,1) = 28.6D0  
MASS(3,2) = 28.6D0

\* INPUT LOCATION OF LINK CENTERS OF GRAVITY

120 LCOGX(1) = 0.0D0  
X1 = LCOGX(1)  
LCOGY(1) = 0.0D0  
Y1 = LCOGY(1)  
LCOGZ(1) = 0.10D0  
Z1 = LCOGZ(1)  
LCOGX(2) = 0.0D0  
X2 = LCOGX(2)  
LCOGY(2) = 0.208D0  
Y2 = LCOGY(2)  
LCOGZ(2) = 0.20D0  
Z2 = LCOGZ(2)  
LCOGX(3) = 0.0D0  
X3 = LCOGX(3)  
LCOGY(3) = 0.6415D0  
Y3 = LCOGY(3)  
LCOGZ(3) = 0.2D0  
Z3 = LCOGZ(3)

\* INPUT MASS OF EACH LINK IN KG

M1 = 44.0D0  
M2 = 4.4D0  
M3 = 57.2D0

\* INPUT ACCELERATIONS OF JOINT ZERO

AX0 = 0.0D0  
AY0 = 0.0D0  
AZ0 = 0.0D0

\* INPUT THE INITIAL DIRECTION COSINES

DRCSX(1) = 0.0D0  
DRCSY(1) = 0.0D0  
DRCSZ(1) = 1.0D0  
DRCSX(2) = 0.0D0  
DRCSY(2) = 1.0D0  
DRCSZ(2) = 0.0D0  
DRCSX(3) = 0.0D0  
DRCSY(3) = 1.0D0  
DRCSZ(3) = 0.0D0

\* INPUT THE INITIAL DIRECTION COSINE ANGLES

DRCANX(1) = 90.0D0  
DRCANY(1) = 90.0D0  
DRCANZ(1) = 0.0D0



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        DRCANX(2) = 90.0D0
        DRCANY(2) = 0.0D0
        DRCANZ(2) = 90.0D0
        DRCANX(3) = 90.0D0
        DRCANY(3) = 0.0D0
        DRCANZ(3) = 90.0D0

*****  INITIALIZE  *****
*      OMEGA AND OMEGA DOT
160      DO 170 I = 1,3
           W1(I)      = 0.0D0
           W2(I)      = 0.0D0
           W3(I)      = 0.0D0
           WDX(I)     = 0.0D0
           WDY(I)     = 0.0D0
           WDZ(I)     = 0.0D0
170      CONTINUE

        THZ = 0.010

*      INITIALIZE MATRIX A AND B TO ZERO
        DO 180 I = 1,27
            DO 175 J = 1,27
                MATA(I,J) = 0.0D0
175      CONTINUE
            MATB(I) = 0.0D0
180      CONTINUE

*****  CALCULATIONS  *****
*      WEIGHTS (NEWTONS)
185      WG1 = M1*G
           WG2 = M2*G
           WG3 = M3*G

*      COMPUTE THE LENGTH FROM THE INBOARD JOINT TO COG
        LNCOG1 = DSQRT ( LCOGX(1)*LCOGX(1) + LCOGY(1)*LCOGY(1) +...
                        LCOGZ(1)*LCOGZ(1) )
        LX2 = LCOGX(2) - JX1
        LY2 = LCOGY(2) - JY1
        LZ2 = LCOGZ(2) - JZ1
        LNCOG2 = DSQRT ( LX2*LX2 + LY2*LY2 + LZ2*LZ2 )
        LX3 = LCOGX(3) - JX2
        LY3 = LCOGY(3) - JY2
        LZ3 = LCOGZ(3) - JZ2
        LNCOG3 = DSQRT ( LX3*LX3 + LY3*LY3 + LZ3*LZ3 )

*      COMPUTE INITIAL INERTIAS BASED ON POINT MASSES
*      IN GLOBAL COORDINATES
190      DO 225 I = 1,3
           RX(I,1) = -L(I,1) * DRCSX(I)
           RX(I,2) =  L(I,2) * DRCSX(I)
           RY(I,1) = -L(I,1) * DRCSY(I)
           RY(I,2) =  L(I,2) * DRCSY(I)
           RZ(I,1) = -L(I,1) * DRCSZ(I)
           RZ(I,2) =  L(I,2) * DRCSZ(I)
200      IXX(I,1) = MASS(I,1) * ((RY(I,1) * RY(I,1)) + (RZ(I,1) * RZ(I,1)))

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      IXX(I,2) = MASS(I,2) * ((RY(I,2) * RY(I,2)) + (RZ(I,2) * RZ(I,2)))
      IXXT(I)  = IXX(I,1) + IXX(I,2)
      IYY(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RZ(I,1) * RZ(I,1)))
      IYY(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RZ(I,2) * RZ(I,2)))
      IYYT(I)  = IYY(I,1) + IYY(I,2)
      IZZ(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RY(I,1) * RY(I,1)))
      IZZ(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RY(I,2) * RY(I,2)))
      IZZT(I)  = IZZ(I,1) + IZZ(I,2)
205  IXY(I,1) = MASS(I,1) * RX(I,1) * RY(I,1)
      IXY(I,2) = MASS(I,2) * RX(I,2) * RY(I,2)
      IXYT(I)  = IXY(I,1) + IXY(I,2)
      IXZ(I,1) = MASS(I,1) * RZ(I,1) * RX(I,1)
      IXZ(I,2) = MASS(I,2) * RZ(I,2) * RX(I,2)
      IXZT(I)  = IXZ(I,1) + IXZ(I,2)
      IYZ(I,1) = MASS(I,1) * RY(I,1) * RZ(I,1)
      IYZ(I,2) = MASS(I,2) * RY(I,2) * RZ(I,2)
      IYZT(I)  = IYZ(I,1) + IYZ(I,2)
      IF (IXXT(I) .LE. .02) THEN
      IXXT(I)  = .02
      ELSE
      IXXT(I) = IXXT(I)
      END IF
      IF (IYYT(I) .LE. .02) THEN
      IYYT(I)  = .02
      ELSE
      IYYT(I)  = IYYT(I)
      END IF
      IF (IZZT(I) .LE. .02) THEN
      IZZT(I)  = .02
      ELSE
      IZZT(I)  = IZZT(I)
      END IF
      IMAT(I,1,1) = IXXT(I)
      IMAT(I,1,2) = IXYT(I)
      IMAT(I,1,3) = IXZT(I)
      IMAT(I,2,1) = -IXYT(I)
      IMAT(I,2,2) = IYYT(I)
      IMAT(I,2,3) = IYZT(I)
      IMAT(I,3,1) = -IXZT(I)
      IMAT(I,3,2) = -IYZT(I)
      IMAT(I,3,3) = IZZT(I)
225  CONTINUE

*      DUE TO LINK 1 CONSTRAINTS LINK 1 INERTIAS ARE CONSTANT
      IXXT(1) = IMAT(1,1,1)
      IXYT(1) = IMAT(1,1,2)
      IXZT(1) = IMAT(1,1,3)
      IYYT(1) = IMAT(1,2,2)
      IYZT(1) = IMAT(1,2,3)
      IZZT(1) = IMAT(1,3,3)

*      TRANSFORM THE INITIAL INERTIAS TO LOCAL COORDINATED
      DO 9 I = 2, 3
      TMat(2,1) = -DCOS ( THZ )
      TMat(2,2) = -DSIN ( THZ )
      TMat(2,3) = 0.0D0

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    TMAT(3,1) = DRCSX(I)
    TMAT(3,2) = DRCSY(I)
    TMAT(3,3) = DRCSZ(I)

    VECTA(1) = TMAT(2,1)
    VECTA(2) = TMAT(2,2)
    VECTA(3) = TMAT(2,3)
    VECTB(1) = TMAT(3,1)
    VECTB(2) = TMAT(3,2)
    VECTB(3) = TMAT(3,3)
    CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
    TMAT(1,1) = MI1
    TMAT(1,2) = MJ1
    TMAT(1,3) = MK1
    TMATTR(1,1) = TMAT(1,1)
    TMATTR(1,2) = TMAT(2,1)
    TMATTR(1,3) = TMAT(3,1)
    TMATTR(2,1) = TMAT(1,2)
    TMATTR(2,2) = TMAT(2,2)
    TMATTR(2,3) = TMAT(3,2)
    TMATTR(3,1) = TMAT(1,3)
    TMATTR(3,2) = TMAT(2,3)
    TMATTR(3,3) = TMAT(3,3)

    DO 5 I1 = 1,3
    DO 5 J = 1,3
        TEMP = 0.0D0
        DO 1 K = 1,3
            TEMP = TMAT(I1,K) * IMAT(I,K,J) + TEMP
1        CONTINUE
        MATTMP(I1,J) = TEMP
5        CONTINUE

    DO 8 I1 = 1,3
    DO 8 J = 1,3
        TEMP = 0.0D0
        DO 7 K = 1,3
            TEMP = MATTMP(I1,K) * TMATTR(K,J) + TEMP
7        CONTINUE
        LIMAT(I,I1,J) = TEMP
8        CONTINUE
9        CONTINUE

DERIVATIVE
NOSORT
230      CALL ERRSET (208,256,-1,1,1)
        CALL UERSET(LEVELQ,LEVLDQ)
*      INITIALIZE MATRIX A AND B TO ZERO
        DO 240 I = 1,27
            DO 235 J = 1,27
                MATA(I,J) = 0.0D0
235          CONTINUE
            MATB(I) = 0.0D0
240          CONTINUE

*      INPUT JOINT EQUATIONS

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*      JOINT ZERO EQUATIONS
*      W1 X (W1 X RB/G1)
          RBG1(1) = JX0 - LCOGX(1)
          RBG1(2) = JY0 - LCOGY(1)
          RBG1(3) = JZ0 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD(VECTA0,RBG1,MICO,MJCO,MKCO)
          VECTB0(1) = MICO
          VECTB0(2) = MJCO
          VECTB0(3) = MKCO
          CALL CPROD(VECTA0,VECTB0,MICO,MJCO,MKCO)

*      INPUT JOINT EQUATIONS
*      JOINT ZERO EQUATIONS
*      W1 X (W1 X RB/G1)
          RBG1(1) = JX0 - LCOGX(1)
          RBG1(2) = JY0 - LCOGY(1)
          RBG1(3) = JZ0 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD(VECTA,RBG1,MICO,MJCO,MKCO)
          VECTB(1) = MICO
          VECTB(2) = MJCO
          VECTB(3) = MKCO
          CALL CPROD(VECTA,VECTB,MICO,MJCO,MKCO)

*      W1 X (W1 X RA/G1)
          RAG1(1) = JX1 - LCOGX(1)
          RAG1(2) = JY1 - LCOGY(1)
          RAG1(3) = JZ1 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD (VECTA,RAG1,MIC1,MJC1,MKC1)
          VECTB(1) = MIC1
          VECTB(2) = MJC1
          VECTB(3) = MKC1
          CALL CPROD (VECTA,VECTB,MIC1,MJC1,MKC1)

*      W2 X (W2 X RB/G2)
          RBG2(1) = JX1 - LCOGX(2)
          RBG2(2) = JY1 - LCOGY(2)
          RBG2(3) = JZ1 - LCOGZ(2)
          VECTA(1) = W2(1)
          VECTA(2) = W2(2)
          VECTA(3) = W2(3)
          CALL CPROD (VECTA,RBG2,MIC2,MJC2,MKC2)
          VECTB(1) = MIC2
          VECTB(2) = MJC2
          VECTB(3) = MKC2
          CALL CPROD (VECTA,VECTB,MIC2,MJC2,MKC2)

*      W2 X (W2 X RA/G2)
          RAG2(1) = JX2 - LCOGX(2)
          RAG2(2) = JY2 - LCOGY(2)

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      RAG2(3) = JZ2 - LCOGZ(2)
      VECTA(1) = W2(1)
      VECTA(2) = W2(2)
      VECTA(3) = W2(3)
      CALL CPROD (VECTA,RAG2,MIC3,MJC3,MKC3)
      VECTB(1) = MIC3
      VECTB(2) = MJC3
      VECTB(3) = MKC3
      CALL CPROD(VECTA,VECTB,MIC3,MJC3,MKC3)
*
W3 X (W3 X RB/G3)
      RBG3(1) = JX2 - LCOGX(3)
      RBG3(2) = JY2 - LCOGY(3)
      RBG3(3) = JZ2 - LCOGZ(3)
      VECTA(1) = W3(1)
      VECTA(2) = W3(2)
      VECTA(3) = W3(3)
      CALL CPROD (VECTA,RBG3,MIC4,MJC4,MKC4)
      VECTB(1) = MIC4
      VECTB(2) = MJC4
      VECTB(3) = MKC4
      CALL CPROD (VECTA,VECTB,MIC4,MJC4,MKC4)

*
INERTIA TRANSFORMATION
DO 390 I = 2, 3
      TMAT(2,1) = -DCOS ( THZ )
      TMAT(2,2) = -DSIN ( THZ )
      TMAT(2,3) = 0.0D0
      TMAT(3,1) = DRCSX(I)
      TMAT(3,2) = DRCSY(I)
      TMAT(3,3) = DRCSZ(I)

      VECTA(1) = TMAT(2,1)
      VECTA(2) = TMAT(2,2)
      VECTA(3) = TMAT(2,3)
      VECTB(1) = TMAT(3,1)
      VECTB(2) = TMAT(3,2)
      VECTB(3) = TMAT(3,3)
      CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
      TMAT(1,1) = MI1
      TMAT(1,2) = MJ1
      TMAT(1,3) = MK1
      TMATTR(1,1) = TMAT(1,1)
      TMATTR(1,2) = TMAT(2,1)
      TMATTR(1,3) = TMAT(3,1)
      TMATTR(2,1) = TMAT(1,2)
      TMATTR(2,2) = TMAT(2,2)
      TMATTR(2,3) = TMAT(3,2)
      TMATTR(3,1) = TMAT(1,3)
      TMATTR(3,2) = TMAT(2,3)
      TMATTR(3,3) = TMAT(3,3)

      DO 375 I1 = 1,3
      DO 375 J = 1,3
          TEMP = 0.0D0
          DO 370 K = 1,3

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      TEMP = TMATTR(I1,K) * LIMAT(I,K,J) + TEMP
370      CONTINUE
      MATTMP(I1,J) = TEMP
375      CONTINUE

      DO 380 I1 = 1,3
      DO 380 J = 1,3
      TEMP = 0.0D0
      DO 378 K = 1,3
      TEMP = MATTMP(I1,K) * TMAT(K,J) + TEMP
378      CONTINUE
      NIMAT(I1,J) = TEMP
380      CONTINUE

      IXXT(I) = NIMAT(1,1)
      IXYT(I) = NIMAT(1,2)
      IXZT(I) = NIMAT(1,3)
      IYYT(I) = NIMAT(2,2)
      IYZT(I) = NIMAT(2,3)
      IZZT(I) = NIMAT(3,3)

390      CONTINUE

***** ENTER CONSTANTS INTO MATRIX A AND B *****
** LINK ONE
* SUM OF FORCES
* X DIRECTION
395      MATA(1,1) = 1.0D0
      MATA(1,4) = M1
      MATA(1,10) = -1.0D0
      MATB(1) = 0.0D0
* Y DIRECTION
      MATA(2,2) = 1.0D0
      MATA(2,5) = M1
      MATA(2,11) = -1.0D0
      MATB(2) = 0.0D0
* Z DIRECTION
      MATA(3,3) = 1.0D0
      MATA(3,6) = M1
      MATA(3,12) = -1.0D0
      MATB(3) = -WG1
* EQUATIONS AT JOINT ZERO
* X DIRECTION
      MATA(4,4) = 1.0D0
      MATA(4,8) = RBG1(3)
      MATA(4,9) = -RBG1(2)
      MATB(4) = AX0 - MICO
* Y DIRECTION
      MATA(5,5) = 1.0D0
      MATA(5,7) = -RBG1(3)
      MATA(5,9) = RBG1(1)
      MATB(5) = AYO - MJCO
* Z DIRECTION
      MATA(6,6) = 1.0D0
      MATA(6,7) = RBG1(2)
      MATA(6,8) = -RBG1(1)

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      MATB(6) = AZO - MKCO
* SUM OF MOMENTS EQUATIONS
* X DIRECTION
      MATA(7,2) = RBG1(3)
      MATA(7,3) = -RBG1(2)
      MATA(7,7) = -IXXT(1)
      MATA(7,8) = IXYT(1)
      MATA(7,9) = IXZT(1)
      MATA(7,11) = -RAG1(3)
      MATA(7,12) = RAG1(2)
      MATB(7) = T1X - TOX
* Y DIRECTION
      MATA(8,1) = -RBG1(3)
      MATA(8,3) = RBG1(1)
      MATA(8,7) = IXYT(1)
      MATA(8,8) = -IYYT(1)
      MATA(8,9) = IYZT(1)
      MATA(8,10) = RAG1(3)
      MATA(8,12) = -RAG1(1)
      MATB(8) = T1Y - TOY
* Z DIRECTION
      MATA(9,1) = RBG1(2)
      MATA(9,2) = -RBG1(1)
      MATA(9,7) = IXZT(1)
      MATA(9,8) = IYZT(1)
      MATA(9,9) = -IZZT(1)
      MATA(9,10) = -RAG1(2)
      MATA(9,11) = RAG1(1)
      MATB(9) = T1Z - TOZ
** LINK TWO
* SUM OF FORCES
* X DIRECTION
460      MATA(10,10) = 1.0D0
      MATA(10,13) = M2
      MATA(10,19) = -1.0D0
      MATB(10) = 0.0D0
* Y DIRECTION
      MATA(11,11) = 1.0D0
      MATA(11,14) = M2
      MATA(11,20) = -1.0D0
      MATB(11) = 0.0D0
* Z DIRECTION
      MATA(12,12) = 1.0D0
      MATA(12,15) = M2
      MATA(12,21) = -1.0D0
      MATB(12) = -WG2
* EQUATIONS AT JOINT ONE
* X DIRECTION
      MATA(13,4) = -1.0D0
      MATA(13,8) = -RAG1(3)
      MATA(13,9) = RAG1(2)
      MATA(13,13) = 1.0D0
      MATA(13,17) = RBG2(3)
      MATA(13,18) = -RBG2(2)
      MATB(13) = MIC1 - MIC2
* Y DIRECTION

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      MATA(14,5) = -1.0D0
      MATA(14,7) = RAG1(3)
      MATA(14,9) = -RAG1(1)
      MATA(14,14) = 1.0D0
      MATA(14,16) = -RBG2(3)
      MATA(14,18) = RBG2(1)
      MATB(14) = MJC1 - MJC2
*      Z DIRECTION
      MATA(15,6) = -1.0D0
      MATA(15,7) = -RAG1(2)
      MATA(15,8) = RAG1(1)
      MATA(15,15) = 1.0D0
      MATA(15,16) = RBG2(2)
      MATA(15,17) = -RBG2(1)
      MATB(15) = MKC1 - MKC2
*      SUM OF MOMENTS EQUATIONS
*      X DIRECTION
      MATA(16,11) = RBG2(3)
      MATA(16,12) = -RBG2(2)
      MATA(16,16) = -IXXT(2)
      MATA(16,17) = IXYT(2)
      MATA(16,18) = IXZT(2)
      MATA(16,20) = -RAG2(3)
      MATA(16,21) = RAG2(2)
      MATB(16) = T2X - T1X
*      Y DIRECTION
      MATA(17,10) = -RBG2(3)
      MATA(17,12) = RBG2(1)
      MATA(17,16) = IXYT(2)
      MATA(17,17) = -IYYT(2)
      MATA(17,18) = IYZT(2)
      MATA(17,19) = RAG2(3)
      MATA(17,21) = -RAG2(1)
      MATB(17) = T2Y - T1Y
*      Z DIRECTION
      MATA(18,10) = RBG2(2)
      MATA(18,11) = -RBG2(1)
      MATA(18,16) = IXZT(2)
      MATA(18,17) = IYZT(2)
      MATA(18,18) = -IZZT(2)
      MATA(18,19) = -RAG2(2)
      MATA(18,20) = RAG2(1)
      MATB(18) = T2Z - T1Z
**      LINK THREE
*      SUM OF FORCES
*      X DIRECTION
530      MATA(19,19) = 1.0D0
      MATA(19,22) = M3
      MATB(19) = 0.0D0
*      Y DIRECTION
      MATA(20,20) = 1.0D0
      MATA(20,23) = M3
      MATB(20) = 0.0D0
*      Z DIRECTION
      MATA(21,21) = 1.0D0
      MATA(21,24) = M3

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      MATB(21) = -WG3
*      EQUATIONS AT JOINT TWO
*      X DIRECTION
      MATA(22,13) = -1.0D0
      MATA(22,17) = -RAG2(3)
      MATA(22,18) =  RAG2(2)
      MATA(22,22) =  1.0D0
      MATA(22,26) =  RBG3(3)
      MATA(22,27) = -RBG3(2)
      MATB(22)   =  MIC3 - MIC4
*      Y DIRECTION
      MATA(23,14) = -1.0D0
      MATA(23,16) =  RAG2(3)
      MATA(23,18) = -RAG2(1)
      MATA(23,23) =  1.0D0
      MATA(23,25) = -RBG3(3)
      MATA(23,27) =  RBG3(1)
      MATB(23)   =  MJC3 - MJC4
*      Z DIRECTION
      MATA(24,15) = -1.0D0
      MATA(24,16) = -RAG2(2)
      MATA(24,17) =  RAG2(1)
      MATA(24,24) =  1.0D0
      MATA(24,25) =  RBG3(2)
      MATA(24,26) = -RBG3(1)
      MATB(24)   =  MKC3 - MKC4
*      SUM OF MOMENTS EQUATIONS
*      X DIRECTION
      MATA(25,20) =  RBG3(3)
      MATA(25,21) = -RBG3(2)
      MATA(25,25) = -IXXT(3)
      MATA(25,26) =  IXYT(3)
      MATA(25,27) =  IXZT(3)
      MATB(25)   = -T2X
*      Y DIRECTION
      MATA(26,19) = -RBG3(3)
      MATA(26,21) =  RBG3(1)
      MATA(26,25) =  IXYT(3)
      MATA(26,26) = -IYYT(3)
      MATA(26,27) =  IYZT(3)
      MATB(26)   = -T2Y
*      Z DIRECTION
      MATA(27,19) =  RBG3(2)
      MATA(27,20) = -RBG3(1)
      MATA(27,25) =  IXZT(3)
      MATA(27,26) =  IYZT(3)
      MATA(27,27) = -IZZT(3)
      MATB(27)   = -T2Z

***** FIRST LINK IS CONSTRAINED TO ROTATE IN Z DIRECTION *****
*****
590      DO 600 I = 4,8
          DO 595 J = 1,27
              MATA(I,J) = 0.0D0
*              MATA(J,I) = 0.0D0
595      CONTINUE

```

```

        MATB(I) = 0.0D0
600    CONTINUE
        MATA(4,4) = 1.0D0
        MATA(5,5) = 1.0D0
        MATA(6,6) = 1.0D0
        MATA(7,7) = 1.0D0
        MATA(8,8) = 1.0D0

605    DO 610 J = 1,27
        MATA(18,J) = 0.0D0
        MATA(27,J) = 0.0D0
610    CONTINUE

        MATA(9,9) = -(IZZT(1)+IZZT(2)+IZZT(3))
        MATA(18,9) = -1.0D0
        MATA(18,18) = 1.0D0
        MATB(18) = 0.0D0
        MATA(27,9) = -1.0D0
        MATA(27,27) = 1.0D0
        MATB(27) = 0.0D0

*      CALL EQUATION SOLVER PROGRAM FROM IMSL
620    CALL LEQT2F(MATA,M,N,IA,MATB,IDGT,WKAREA,IER)
        IF (IER.EQ.0) THEN
            GOTO 640
        ELSE
            WRITE (*,624) IER
624    FORMAT (I5)
            DO 635 I = 1, 27
            WRITE (*,627) I
627    FORMAT (I7)
            DO 631 J = 1, 27, 3
            WRITE (*,630) J,MATA(I,J),J+1,MATA(I,J+1),J+2,MATA(I,J+2)
630    FORMAT (I5,F13.5,I5,F13.5,I5,F13.5)
631    CONTINUE
            WRITE (*,633) I,MATB(I)
633    FORMAT (I3,F15.5)
635    CONTINUE
            CALL ENDJOB
            END IF

***    FIND LCOGX,LCOGY,LCOGZ,THETA VALUES,WX,WY,WZ

*      JOINT ZERO
640    FX0 = MATB(1)
        FY0 = MATB(2)
        FZ0 = MATB(3)

*      LINK ONE
*      SINCE LINK1 IS CONSTRAIN TO ROTATE AROUND THE Z ONLY
        AX1 = 0.0D0
        AY1 = 0.0D0
        AZ1 = 0.0D0
*660    AX1 = MATB(4)
*      VELX1 = INTGRL(0.,AX1)
*      LCOGX1 = INTGRL(X1,VELX1)
*      LCOGX(1) = LCOGX1

```

```

*      AY1      = MATB(5)
*      VELY1     = INTGRL(0.,AY1)
*      LCOGY1    = INTGRL(Y1,VELY1)
*      LCOGY(1)  = LCOGY1
*      AZ1       = MATB(6)
*      VELZ1     = INTGRL(0.,AZ1)
*      LCOGZ1    = INTGRL(Z1,VELZ1)
*      LCOGZ(1)  = LCOGZ1
WD1X      = MATB(7)
W1X       = INTGRL(0.,WD1X)
WDX(1)    = WD1X
W1(1)     = W1X
WD1Y      = MATB(8)
W1Y       = INTGRL(0.,WD1Y)
WDY(1)    = WD1Y
W1(2)     = W1Y
WD1Z      = MATB(9)
W1Z       = INTGRL(0.,WD1Z)
WDZ(1)    = WD1Z
W1(3)     = W1Z
***      ADDED BY R.M. VERBOS
685      THZ     = INTGRL(0.,W1Z)
          THZANG = THZ * RADEG
          COSTHZ = DCOS(THZ)
          SINTHZ = DSIN(THZ)

*      IF THE 1ST LINK IS CONSTRAIN TO ROTATE IN THE Z DIRECTION ONLY
*      THE DIRECTION COSINE AND DIRECTION COSINE ANGLES ARE CONSTANT
*      AND DO NOT NEED TO BE CALCULATED
*
*      CALC DIRECTIONAL COSINES FOR LINK ONE
*
*700      DRCSX(1) = LCOGX1 / LNCOG1
*      DRCSY(1) = LCOGY1 / LNCOG1
*      DRCSZ(1) = LCOGZ1 / LNCOG1
*      AMP = DSQRT(DRCSX(1)*DRCSX(1)+DRCSY(1)*DRCSY(1)+...
*              DRCSZ(1)*DRCSZ(1))
*      DRCSX(1) = DRCSX(1)/AMP
*      DRCSY(1) = DRCSY(1)/AMP
*      DRCSZ(1) = DRCSZ(1)/AMP
*720      DRCANX(1) = DACOS(DRCSX(1)) * RADEG
*      DRCANY(1) = DACOS(DRCSY(1)) * RADEG
*      DRCANZ(1) = DACOS(DRCSZ(1)) * RADEG
*
**      JOINT LOCATION
*730      JX1 = LINKL1 * DRCSX(1)
*      JY1 = LINKL1 * DRCSY(1)
*      JZ1 = LINKL1 * DRCSZ(1)

*      JOINT ONE
735      FX1 = MATB(10)
          FY1 = MATB(11)
          FZ1 = MATB(12)
**      LINK TWO
740      AX2 = MATB(13)
          VELX2 = INTGRL(0.,AX2)

```



```

      LCOGX2      = INTGRL(X2,VELX2)
      LCOGX(2)    = LCOGX2
      AY2         = MATB(14)
      VELY2       = INTGRL(0.,AY2)
      LCOGY2      = INTGRL(Y2,VELY2)
      LCOGY(2)    = LCOGY2
      AZ2         = MATB(15)
      VELZ2       = INTGRL(0.,AZ2)
      LCOGZ2      = INTGRL(Z2,VELZ2)
      LCOGZ(2)    = LCOGZ2
      WD2X        = MATB(16)
      W2X         = INTGRL(0.,WD2X)
      WDX(2)      = WD2X
      W2(1)       = W2X
      WD2Y        = MATB(17)
      W2Y         = INTGRL(0.,WD2Y)
      WDY(2)      = WD2Y
      W2(2)       = W2Y
      WD2Z        = MATB(18)
      W2Z         = INTGRL(0.,WD2Z)
      WDZ(2)      = WD2Z
      W2(3)       = W2Z

*      GET THE DIRECTION COSINES FOR THE LINK TWO
      DRCSX(2)    = (LCOGX2 - JX1) / LNCOG2
      DRCSY(2)    = (LCOGY2 - JY1) / LNCOG2
      DRCSZ(2)    = (LCOGZ2 - JZ1) / LNCOG2
790    AMP = DSQRT(DRCSX(2)*DRCSX(2)+DRCSY(2)*DRCSY(2)+...
          DRCSZ(2)*DRCSZ(2))
      DRCSX(2)    = DRCSX(2)/AMP
      DRCSY(2)    = DRCSY(2)/AMP
      DRCSZ(2)    = DRCSZ(2)/AMP
      DRCANX(2)   = DACOS(DRCSX(2)) * RADEG
      DRCANY(2)   = DACOS(DRCSY(2)) * RADEG
      DRCANZ(2)   = DACOS(DRCSZ(2)) * RADEG

*      JOINT LOCATION
800    JX2 = JX1 + LINKL2 * DRCSX(2)
      JY2 = JY1 + LINKL2 * DRCSY(2)
      JZ2 = JZ1 + LINKL2 * DRCSZ(2)

*      JOINT TWO
805    FX2 = MATB(19)
      FY2 = MATB(20)
      FZ2 = MATB(21)

**     LINK THREE
812    AX3      = MATB(22)
      VELX3     = INTGRL(0.,AX3)
      LCOGX3    = INTGRL(X3,VELX3)
      LCOGX(3)  = LCOGX3
      AY3       = MATB(23)
      VELY3     = INTGRL(0.,AY3)
      LCOGY3    = INTGRL(Y3,VELY3)
      LCOGY(3)  = LCOGY3
      AZ3       = MATB(24)
      VELZ3     = INTGRL(0.,AZ3)

```

```

LCOGZ3      = INTGRL(Z3,VELZ3)
LCOGZ(3)    = LCOGZ3
WD3X        = MATB(25)
W3X         = INTGRL(0.,WD3X)
WDX(3)      = WD3X
W3(1)       = W3X
WD3Y        = MATB(26)
W3Y         = INTGRL(0.,WD3Y)
WDY(3)      = WD3Y
W3(2)       = W3Y
WD3Z        = MATB(27)
W3Z         = INTGRL(0.,WD3Z)
WDZ(3)      = WD3Z
W3(3)       = W3Z

```

\* CALC DIRECTIONAL COSINES FOR LINK THREE

```

845      DRCSX(3) = (LCOGX3 - JX2) / LNCOG3
          DRCSY(3) = (LCOGY3 - JY2) / LNCOG3
          DRCSZ(3) = (LCOGZ3 - JZ2) / LNCOG3
865      AMP = DSQRT(DRCSX(3)*DRCSX(3)+DRCSY(3)*DRCSY(3)+...
                  DRCSZ(3)*DRCSZ(3))
          DRCSX(3) = DRCSX(3)/AMP
          DRCSY(3) = DRCSY(3)/AMP
          DRCSZ(3) = DRCSZ(3)/AMP
          DRCANX(3) = DACOS(DRCSX(3)) * RADEG
          DRCANY(3) = DACOS(DRCSY(3)) * RADEG
          DRCANZ(3) = DACOS(DRCSZ(3)) * RADEG

```

\* TIP LOCATION

```

875      TIPX = JX2 + LINKL3 * DRCSX(3)
          TIPY = JY2 + LINKL3 * DRCSY(3)
          TIPZ = JZ2 + LINKL3 * DRCSZ(3)

```

\* FIND THE ANGLE BETWEEN THE LINKS

```

880      ANG23X = DRCANX(2) - DRCANX(3)
          ANG23Y = DRCANY(2) - DRCANY(3)
          ANG23Z = DRCANZ(2) - DRCANZ(3)

          ANG12X = DRCANX(1) - DRCANX(2)
          ANG12Y = DRCANY(1) - DRCANY(2)
          ANG12Z = DRCANZ(1) - DRCANZ(2)

```

DYNAMIC

NOSORT

\*\*\*\*\* INPUT TORQUE AT JOINTS

\* CALCULATE THE MAGNITUDE OF THE LENGTH OF THE PROJECTION OF  
 \* LINK 2 AND 3 IN THE X Y PLANE WHICH IS THE MOMENT ARM FOR  
 \* THE CALCULATION OF THE GRAVITATIONAL TORQUE

```

900      MARM2A = DSQRT ( LCOGX2 * LCOGX2 + LCOGY2 * LCOGY2 )
          MARM2B = DSQRT ( LCOGX3 * LCOGX3 + LCOGY3 * LCOGY3 )
          MARM3  = DSQRT ( (LCOGX3 - JX2) * (LCOGX3 - JX2) +...
                          (LCOGY3 - JY2) * (LCOGY3 - JY2) )

```

\* CALCULATE GRAVITATIONAL TORQUES

```

      TG1 = MARM2A * WG2 + MARM2B * WG3
      TG2 = MARM3 * WG3

*   INPUT TORQUES OVERTOP OF GRAVITY
*10      T1FNC = 2*A - A * DCOS ( W * TIME )
*      T2FNC = -3 * A * DSIN (W*TIME)
*      T0Z   = A * DSIN ( W * TIME )
*      T0Z   = A - A * DSIN ( W * TIME )
*      T0Z   = 50.0 - 50.0 * TIME

*   CALCULATE TOTAL TORQUE ON JOINT 1 AND 2
      T1 = TG1 + T1FNC
      T2 = TG2 + T2FNC

*   CALCULATE X AND Y COMPONENT OF TORQUE ON JOINT 1 AND 2
      T1X = T1 * DCOS (THZ)
      T1Y = T1 * DSIN (THZ)
      T2X = T2 * COS (THZ)
      T2Y = T2 * SIN (THZ)

*   TORQUE CHECK
      T1CH = DSQRT ( T1X * T1X + T1Y * T1Y )
      T2CH = DSQRT ( T2X * T2X + T2Y * T2Y )

END
STOP

FORTRAN

*   SUBROUTINE TO COMPUTE THE CROSS PRODUCT OF TWO VECTORS

      SUBROUTINE CPROD(VECTA,VECTB,MI,MJ,MK)
940      IMPLICIT REAL*8 (A-Z)
      DIMENSION VECTA(3),VECTB(3)
      MI = VECTA(2) * VECTB(3) - VECTA(3) * VECTB(2)
      MJ = VECTA(3) * VECTB(1) - VECTA(1) * VECTB(3)
      MK = VECTA(1) * VECTB(2) - VECTA(2) * VECTB(1)
      RETURN

      END

```

## APPENDIX B

### DOUBLE PENDULUM VALIDATION PROGRAM

TITLE DOUBLE PENDULUM VALIDATION PROGRAM

```
*****
* THIS IS A PROGRAM THAT WILL SOLVE THE DIRECT DYNAMICS PROBLEM FOR A *
* 2 LINK RIGID BODY MANIPULATOR WITHOUT THE USE OF THE STANDARD TRAN- *
* FORMATION MATRICES AND VALIDATE THE MOTION BY THE USE OF THE DOUBLE *
* PENDULUM PROBLEM. *
* LT R. M. VERBOS USN *
*****
```

TERMINAL

METHOD ADAMS

PRINT .10, ANGTH2, TH2ANG, ANGTH3, TH3ANG, ANG23

CONTROL FINTIM = 4.0, DELT = .0005, DELMAX = .1, DELPRT = .10

SAVE .05, ANGTH2, TH2ANG, ANGTH3, TH3ANG, ANG23

GRAPH (DE=TEK618) TIME, ANGTH2, TH2ANG

GRAPH (DE=TEK618) TIME, ANGTH3, TH3ANG

GRAPH (DE=TEK618) TIME, ANG23

D DIMENSION MATA(27,27), MASS(3,2), L(3,2), RX(3,2), RY(3,2), RZ(3,2)

D DIMENSION IXX(3,2), IXZ(3,2), IXY(3,2), IYY(3,2), IYZ(3,2), IZZ(3,2)

D DIMENSION IMAT(3,3,3), NIMAT(3,3), TMAT(3,3), TMATTR(3,3), MATTMP(3,3)

D DIMENSION LIMAT(3,3,3)

FIXED IER, I, J, M, K, P, N, IA, IDGT, A, I1

ARRAY MATB(27), ABMATB(27), LCOGX(3), LCOGY(3), LCOGZ(3)

ARRAY VECTAO(3), VECTBO(3), VECTA1(3), VECTB1(3), VECTA2(3), VECTB2(3)

ARRAY VECTA(3), VECTB(3)

ARRAY WDX(3), WDY(3), WDX(3), W1(3), W2(3), W3(3)

ARRAY RBG1(3), RAG1(3), RBG2(3), RAG2(3), RBG3(3)

ARRAY WKAREA(850)

ARRAY IXXT(3), IYYT(3), IZZT(3), IXYT(3), IXZT(3), IYZT(3)

ARRAY DRCANX(3), DRCANY(3), DRCANZ(3)

ARRAY DRCSX(3), DRCSY(3), DRCSZ(3)

INITIAL

\*\*\*\*\* INPUT \*\*\*\*\*

\* INPUT PARAMETER CONSTANTS

55 A = 15.0D0

P = 0.0D0

W = 2 \* PI

IDGT = 6

\*\*\* MODIFIED BY RM VERBOS

G = 9.81D0

\* G = 0.0D0

N = 27

```

M = 1
IA = 27
IER = 0
LEVELQ = 0
LEVLDQ = 0

*      INPUT JOINT LOCATIONS IN METERS
      JX0 = 0.0D0
      JY0 = 0.0D0
      JZ0 = 0.0D0
      JX1 = 0.0D0
      JY1 = 0.0D0
      JZ1 = 0.2D0
      JX2 = 0.0D0
      JY2 = 0.416D0
      JZ2 = 0.2D0

*      INPUT TORQUE CONSTANTS
      TOX   = 0.0D0
      TOY   = 0.0D0
      TOZ   = 0.0D0
      T1X   = 0.0D0
      T1Y   = 0.0D0
      T1Z   = 0.0D0
      T2X   = 0.0D0
      T2Y   = 0.0D0
      T2Z   = 0.0D0

*      INPUT DISTANCE FROM CENTER OF LINK TO CENTER OF MASS
*      FOR EACH LINK ENDS
      L(1,1) = 0.05D0
      L(1,2) = 0.15D0
      L(2,1) = 0.20D0
      L(2,2) = 0.216D0
      L(3,1) = 0.225D0
      L(3,2) = 0.226D0

* INPUT THE LINK LENGTHS OF THE ROBOT
      LINKL1 = 0.20D0
      LINKL2 = 0.416D0
      LINKL3 = 0.451D0

*      INPUT MASS AT LINK ENDS IN KILOGRAMS
110      MASS(1,1) = 11.0D0
      MASS(1,2) = 33.0D0
      MASS(2,1) = 2.2D0
      MASS(2,2) = 2.2D0
      MASS(3,1) = 28.6D0
      MASS(3,2) = 28.6D0

*      INPUT LOCATION OF LINK CENTERS OF GRAVITY
120      LCOGX(1) = 0.0D0
      X1         = LCOGX(1)
      LCOGY(1) = 0.0D0
      Y1         = LCOGY(1)
      LCOGZ(1) = 0.10D0

```



```

Z1      = LCOGZ(1)
LCOGX(2) = 0.0D0
X2      = LCOGX(2)
LCOGY(2) = 0.208D0
Y2      = LCOGY(2)
LCOGZ(2) = 0.20D0
Z2      = LCOGZ(2)
LCOGX(3) = 0.0D0
X3      = LCOGX(3)
LCOGY(3) = 0.6415D0
Y3      = LCOGY(3)
LCOGZ(3) = 0.2D0
Z3      = LCOGZ(3)

```

\* INPUT MASS OF EACH LINK IN KG

```

M1 = 44.0D0
M2 = 4.4D0
M3 = 57.2D0

```

\* INPUT ACCELERATIONS OF JOINT ZERO

```

AX0 = 0.0D0
AY0 = 0.0D0
AZ0 = 0.0D0

```

\* INPUT THE INITIAL DIRECTION COSINES

```

DRCSX(1) = 0.0D0
DRCSY(1) = 0.0D0
DRCSZ(1) = 1.0D0
DRCSX(2) = 0.0D0
DRCSY(2) = 1.0D0
DRCSZ(2) = 0.0D0
DRCSX(3) = 0.0D0
DRCSY(3) = 1.0D0
DRCSZ(3) = 0.0D0

```

\* INPUT THE INITIAL DIRECTION COSINE ANGLES

```

DRCANX(1) = 90.0D0
DRCANY(1) = 90.0D0
DRCANZ(1) = 0.0D0
DRCANX(2) = 90.0D0
DRCANY(2) = 0.0D0
DRCANZ(2) = 90.0D0
DRCANX(3) = 90.0D0
DRCANY(3) = 0.0D0
DRCANZ(3) = 90.0D0

```

\*\*\*\*\* INITIALIZE \*\*\*\*\*

\* OMEGA AND OMEGA DOT

```

160 DO 170 I = 1,3
      W1(I) = 0.0D0
      W2(I) = 0.0D0
      W3(I) = 0.0D0
      WDX(I) = 0.0D0
      WDY(I) = 0.0D0
      WDZ(I) = 0.0D0

```

```

170 CONTINUE

```

```

        THZ = 0.0D0

*      INITIALIZE MATRIX A AND B TO ZERO
        DO 180 I = 1,27
            DO 175 J = 1,27
                MATA(I,J) = 0.0D0
175          CONTINUE
            MATB(I) = 0.0D0
180          CONTINUE

*****      CALCULATIONS      *****
*      WEIGHTS (NEWTONS)
185          WG1 = M1*G
            WG2 = M2*G
            WG3 = M3*G

*      COMPUTE THE LENGTH FROM THE INBOARD JOINT TO COG
        LNCOG1 = DSQRT ( LCOGX(1)*LCOGX(1) + LCOGY(1)*LCOGY(1) +...
                        LCOGZ(1)*LCOGZ(1) )
        LX2 = LCOGX(2) - JX1
        LY2 = LCOGY(2) - JY1
        LZ2 = LCOGZ(2) - JZ1
        LNCOG2 = DSQRT ( LX2*LX2 + LY2*LY2 + LZ2*LZ2 )
        LX3 = LCOGX(3) - JX2
        LY3 = LCOGY(3) - JY2
        LZ3 = LCOGZ(3) - JZ2
        LNCOG3 = DSQRT ( LX3*LX3 + LY3*LY3 + LZ3*LZ3 )

*      COMPUTE INITIAL INERTIAS BASED ON POINT MASSES
*      IN GLOBAL COORDINATES
190          DO 225 I = 1,3
                RX(I,1) = -L(I,1) * DRCSX(I)
                RX(I,2) =  L(I,2) * DRCSX(I)
                RY(I,1) = -L(I,1) * DRCSY(I)
                RY(I,2) =  L(I,2) * DRCSY(I)
                RZ(I,1) = -L(I,1) * DRCSZ(I)
                RZ(I,2) =  L(I,2) * DRCSZ(I)
200          IXX(I,1) = MASS(I,1) * ((RY(I,1) * RY(I,1)) + (RZ(I,1) * RZ(I,1)))
            IXX(I,2) = MASS(I,2) * ((RY(I,2) * RY(I,2)) + (RZ(I,2) * RZ(I,2)))
            IXXT(I)  = IXX(I,1) + IXX(I,2)
            IYY(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RZ(I,1) * RZ(I,1)))
            IYY(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RZ(I,2) * RZ(I,2)))
            IYYT(I)  = IYY(I,1) + IYY(I,2)
            IZZ(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RY(I,1) * RY(I,1)))
            IZZ(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RY(I,2) * RY(I,2)))
            IZZT(I)  = IZZ(I,1) + IZZ(I,2)
205          IXY(I,1) = MASS(I,1) * RX(I,1) * RY(I,1)
            IXY(I,2) = MASS(I,2) * RX(I,2) * RY(I,2)
            IXYT(I)  = IXY(I,1) + IXY(I,2)
            IXZ(I,1) = MASS(I,1) * RZ(I,1) * RX(I,1)
            IXZ(I,2) = MASS(I,2) * RZ(I,2) * RX(I,2)
            IXZT(I)  = IXZ(I,1) + IXZ(I,2)
            IYZ(I,1) = MASS(I,1) * RY(I,1) * RZ(I,1)
            IYZ(I,2) = MASS(I,2) * RY(I,2) * RZ(I,2)
            IYZT(I)  = IYZ(I,1) + IYZ(I,2)

```

```

IF (IXXT(I) .LE. .01) THEN
  IXXT(I) = .01
ELSE
  IXXT(I) = IXXT(I)
END IF
IF (IYYT(I) .LE. .02) THEN
  IYYT(I) = .02
ELSE
  IYYT(I) = IYYT(I)
END IF
IF (IZZT(I) .LE. .02) THEN
  IZZT(I) = .02
ELSE
  IZZT(I) = IZZT(I)
END IF
      IMAT(I,1,1) = IXXT(I)
      IMAT(I,1,2) = IXYT(I)
      IMAT(I,1,3) = IXZT(I)
      IMAT(I,2,1) = -IXYT(I)
      IMAT(I,2,2) = IYYT(I)
      IMAT(I,2,3) = IYZT(I)
      IMAT(I,3,1) = -IXZT(I)
      IMAT(I,3,2) = -IYZT(I)
      IMAT(I,3,3) = IZZT(I)
225      CONTINUE

```

```

*      DUE TO LINK 1 CONSTRAINTS LINK 1 INERTIAS ARE CONSTANT
      IXXT(1) = IMAT(1,1,1)
      IXYT(1) = IMAT(1,1,2)
      IXZT(1) = IMAT(1,1,3)
      IYYT(1) = IMAT(1,2,2)
      IYZT(1) = IMAT(1,2,3)
      IZZT(1) = IMAT(1,3,3)

```

```

*      TRANSFORM THE INITIAL INERTIAS TO LOCAL COORDINATED
      DO 9 I = 2, 3
        TMAT(2,1) = -DCOS ( THZ )
        TMAT(2,2) = -DSIN ( THZ )
        TMAT(2,3) = 0.0D0
        TMAT(3,1) = DRCSX(I)
        TMAT(3,2) = DRCSY(I)
        TMAT(3,3) = DRCSZ(I)

        VECTA(1) = TMAT(2,1)
        VECTA(2) = TMAT(2,2)
        VECTA(3) = TMAT(2,3)
        VECTB(1) = TMAT(3,1)
        VECTB(2) = TMAT(3,2)
        VECTB(3) = TMAT(3,3)
        CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
        TMAT(1,1) = MI1
        TMAT(1,2) = MJ1
        TMAT(1,3) = MK1
        TMATTR(1,1) = TMAT(1,1)
        TMATTR(1,2) = TMAT(2,1)
        TMATTR(1,3) = TMAT(3,1)

```

```

      TMATTR(2,1) = TMAT(1,2)
      TMATTR(2,2) = TMAT(2,2)
      TMATTR(2,3) = TMAT(3,2)
      TMATTR(3,1) = TMAT(1,3)
      TMATTR(3,2) = TMAT(2,3)
      TMATTR(3,3) = TMAT(3,3)

      DO 5 I1 = 1,3
      DO 5 J = 1,3
        TEMP = 0.0D0
        DO 1 K = 1,3
          TEMP = TMAT(I1,K) * IMAT(I,K,J) + TEMP
1      CONTINUE
        MATTMP(I1,J) = TEMP
5      CONTINUE

      DO 8 I1 = 1,3
      DO 8 J = 1,3
        TEMP = 0.0D0
        DO 7 K = 1,3
          TEMP = MATTMP(I1,K) * TMATTR(K,J) + TEMP
7      CONTINUE
        LIMAT(I,I1,J) = TEMP
8      CONTINUE
9      CONTINUE

*      DOUBLE PENDULUM INITIALIZATION
      TH2D0 = 0.0D0
      TH3D0 = 0.0D0
      TH30 = PI/2.0
      TH20 = PI/2.0
      L2 = 0.416D0
      L3 = 0.451D0
      M2P = 30.8D0
      M3P = 28.6D0
      HF = 0.5D0

DERIVATIVE
NOSORT
230      CALL ERRSET (208,256,-1,1,1)
      CALL UERSET(LEVELQ,LEVLDQ)

*      INITIALIZE MATRIX A AND B TO ZERO
      DO 240 I = 1,27
        DO 235 J = 1,27
          MATA(I,J) = 0.0D0
235      CONTINUE
        MATB(I) = 0.0D0
240      CONTINUE

*      INPUT JOINT EQUATIONS

*      JOINT ZERO EQUATIONS
*      W1 X (W1 X RB/G1)
*      RBG1(1) = JX0 - LCOGX(1)

```

```

*          RBG1(2) = JY0 - LCOGY(1)
*          RBG1(3) = JZ0 - LCOGZ(1)
*          VECTA(1) = W1(1)
*          VECTA(2) = W1(2)
*          VECTA(3) = W1(3)
*          CALL CPROD(VECTA0,RBG1,MICO,MJCO,MKCO)
*          VECTB0(1) = MICO
*          VECTB0(2) = MJCO
*          VECTB0(3) = MKCO
*          CALL CPROD(VECTA0,VECTB0,MICO,MJCO,MKCO)
**      SINCE LINK ONE DOES NOT MOVE
          MICO = 0.0D0
          MJCO = 0.0D0
          MKCO = 0.0D0

*      INPUT JOINT EQUATIONS
*      JOINT ZERO EQUATIONS
*      W1 X (W1 X RB/G1)
          RBG1(1) = JX0 - LCOGX(1)
          RBG1(2) = JY0 - LCOGY(1)
          RBG1(3) = JZ0 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD(VECTA,RBG1,MICO,MJCO,MKCO)
          VECTB(1) = MICO
          VECTB(2) = MJCO
          VECTB(3) = MKCO
          CALL CPROD(VECTA,VECTB,MICO,MJCO,MKCO)
*      W1 X (W1 X RA/G1)
          RAG1(1) = JX1 - LCOGX(1)
          RAG1(2) = JY1 - LCOGY(1)
          RAG1(3) = JZ1 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD (VECTA,RAG1,MIC1,MJC1,MKC1)
          VECTB(1) = MIC1
          VECTB(2) = MJC1
          VECTB(3) = MKC1
          CALL CPROD (VECTA,VECTB,MIC1,MJC1,MKC1)
*      W2 X (W2 X RB/G2)
          RBG2(1) = JX1 - LCOGX(2)
          RBG2(2) = JY1 - LCOGY(2)
          RBG2(3) = JZ1 - LCOGZ(2)
          VECTA(1) = W2(1)
          VECTA(2) = W2(2)
          VECTA(3) = W2(3)
          CALL CPROD (VECTA,RBG2,MIC2,MJC2,MKC2)
          VECTB(1) = MIC2
          VECTB(2) = MJC2
          VECTB(3) = MKC2
          CALL CPROD (VECTA,VECTB,MIC2,MJC2,MKC2)
*      W2 X (W2 X RA/G2)
          RAG2(1) = JX2 - LCOGX(2)
          RAG2(2) = JY2 - LCOGY(2)

```



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      RAG2(3) = JZ2 - LCOGZ(2)
      VECTA(1) = W2(1)
      VECTA(2) = W2(2)
      VECTA(3) = W2(3)
      CALL CPROD (VECTA,RAG2,MIC3,MJC3,MKC3)
      VECTB(1) = MIC3
      VECTB(2) = MJC3
      VECTB(3) = MKC3
      CALL CPROD(VECTA,VECTB,MIC3,MJC3,MKC3)
*
W3 X (W3 X RB/G3)
      RBG3(1) = JX2 - LCOGX(3)
      RBG3(2) = JY2 - LCOGY(3)
      RBG3(3) = JZ2 - LCOGZ(3)
      VECTA(1) = W3(1)
      VECTA(2) = W3(2)
      VECTA(3) = W3(3)
      CALL CPROD (VECTA,RBG3,MIC4,MJC4,MKC4)
      VECTB(1) = MIC4
      VECTB(2) = MJC4
      VECTB(3) = MKC4
      CALL CPROD (VECTA,VECTB,MIC4,MJC4,MKC4)

*
INERTIA TRANSFORMATION
DO 390 I = 2, 3
  TMAT(2,1) = -DCOS ( THZ )
  TMAT(2,2) = -DSIN ( THZ )
  TMAT(2,3) = 0.0D0
  TMAT(3,1) = DRCSX(I)
  TMAT(3,2) = DRCSY(I)
  TMAT(3,3) = DRCSZ(I)

  VECTA(1) = TMAT(2,1)
  VECTA(2) = TMAT(2,2)
  VECTA(3) = TMAT(2,3)
  VECTB(1) = TMAT(3,1)
  VECTB(2) = TMAT(3,2)
  VECTB(3) = TMAT(3,3)
  CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
  TMAT(1,1) = MI1
  TMAT(1,2) = MJ1
  TMAT(1,3) = MK1
  TMATTR(1,1) = TMAT(1,1)
  TMATTR(1,2) = TMAT(2,1)
  TMATTR(1,3) = TMAT(3,1)
  TMATTR(2,1) = TMAT(1,2)
  TMATTR(2,2) = TMAT(2,2)
  TMATTR(2,3) = TMAT(3,2)
  TMATTR(3,1) = TMAT(1,3)
  TMATTR(3,2) = TMAT(2,3)
  TMATTR(3,3) = TMAT(3,3)

DO 375 I1 = 1,3
DO 375 J = 1,3
  TEMP = 0.0D0
DO 370 K = 1,3

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      TEMP = TMATTR(I1,K) * LIMAT(I,K,J) + TEMP
370      CONTINUE
      MATTMP(I1,J) = TEMP
375      CONTINUE

      DO 380 I1 = 1,3
      DO 380 J = 1,3
      TEMP = 0.0D0
      DO 378 K = 1,3
      TEMP = MATTMP(I1,K) * TMAT(K,J) + TEMP
378      CONTINUE
      NIMAT(I1,J) = TEMP
380      CONTINUE

      IXXT(I) = NIMAT(1,1)
      IXYT(I) = NIMAT(1,2)
      IXZT(I) = NIMAT(1,3)
      IYYT(I) = NIMAT(2,2)
      IYZT(I) = NIMAT(2,3)
      IZZT(I) = NIMAT(3,3)

390      CONTINUE

***** ENTER CONSTANTS INTO MATRIX A *****

** LINK ONE
* SUM OF FORCES
* X DIRECTION
395 MATA(1,1) = 1.0D0
      MATA(1,4) = M1
      MATA(1,10) = -1.0D0
      MATB(1) = 0.0D0
* Y DIRECTION
      MATA(2,2) = 1.0D0
      MATA(2,5) = M1
      MATA(2,11) = -1.0D0
      MATB(2) = 0.0D0
* Z DIRECTION
      MATA(3,3) = 1.0D0
      MATA(3,6) = M1
      MATA(3,12) = -1.0D0
      MATB(3) = -WG1
* EQUATIONS AT JOINT ZERO
* X DIRECTION
      MATA(4,4) = 1.0D0
      MATA(4,8) = RBG1(3)
      MATA(4,9) = -RBG1(2)
      MATB(4) = AX0 - MICO
* Y DIRECTION
      MATA(5,5) = 1.0D0
      MATA(5,7) = -RBG1(3)
      MATA(5,9) = RBG1(1)
      MATB(5) = AY0 - MJCO
* Z DIRECTION
      MATA(6,6) = 1.0D0
      MATA(6,7) = RBG1(2)

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      MATA(6,8) = -RBG1(1)
      MATB(6)   = AZO - MKCO
*
* SUM OF MOMENTS EQUATIONS
* X DIRECTION
      MATA(7,2) = RBG1(3)
      MATA(7,3) = -RBG1(2)
      MATA(7,7) = -IXXT(1)
      MATA(7,8) = IXYT(1)
      MATA(7,9) = IXZT(1)
      MATA(7,11) = -RAG1(3)
      MATA(7,12) = RAG1(2)
      MATB(7)   = T1X - TOX
*
* Y DIRECTION
      MATA(8,1) = -RBG1(3)
      MATA(8,3) = RBG1(1)
      MATA(8,7) = IXYT(1)
      MATA(8,8) = -IYYT(1)
      MATA(8,9) = IYZT(1)
      MATA(8,10) = RAG1(3)
      MATA(8,12) = -RAG1(1)
      MATB(8)   = T1Y - TOY
*
* Z DIRECTION
      MATA(9,1) = RBG1(2)
      MATA(9,2) = -RBG1(1)
      MATA(9,7) = IXZT(1)
      MATA(9,8) = IYZT(1)
      MATA(9,9) = -IZZT(1)
      MATA(9,10) = -RAG1(2)
      MATA(9,11) = RAG1(1)
      MATB(9)   = T1Z - TOZ

** LINK TWO
* SUM OF FORCES
* X DIRECTION
460      MATA(10,10) = 1.0D0
      MATA(10,13) = M2
      MATA(10,19) = -1.0D0
      MATB(10)   = 0.0D0
*
* Y DIRECTION
      MATA(11,11) = 1.0D0
      MATA(11,14) = M2
      MATA(11,20) = -1.0D0
      MATB(11)   = 0.0D0
*
* Z DIRECTION
      MATA(12,12) = 1.0D0
      MATA(12,15) = M2
      MATA(12,21) = -1.0D0
      MATB(12)   = -WG2
*
* EQUATIONS AT JOINT ONE
* X DIRECTION
      MATA(13,4) = -1.0D0
      MATA(13,8) = -RAG1(3)
      MATA(13,9) = RAG1(2)
      MATA(13,13) = 1.0D0
      MATA(13,17) = RBG2(3)
      MATA(13,18) = -RBG2(2)

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      MATB(13)      =  MIC1 - MIC2
*   Y DIRECTION
      MATA(14,5)    =  -1.0D0
      MATA(14,7)    =  RAG1(3)
      MATA(14,9)    =  -RAG1(1)
      MATA(14,14)   =  1.0D0
      MATA(14,16)   =  -RBG2(3)
      MATA(14,18)   =  RBG2(1)
      MATB(14)      =  MJC1 - MJC2
*   Z DIRECTION
      MATA(15,6)    =  -1.0D0
      MATA(15,7)    =  -RAG1(2)
      MATA(15,8)    =  RAG1(1)
      MATA(15,15)   =  1.0D0
      MATA(15,16)   =  RBG2(2)
      MATA(15,17)   =  -RBG2(1)
      MATB(15)      =  MKC1 - MKC2
*   SUM OF MOMENTS EQUATIONS
*   X DIRECTION
      MATA(16,11)   =  RBG2(3)
      MATA(16,12)   =  -RBG2(2)
      MATA(16,16)   =  -IXXT(2)
      MATA(16,17)   =  IXYT(2)
      MATA(16,18)   =  IXZT(2)
      MATA(16,20)   =  -RAG2(3)
      MATA(16,21)   =  RAG2(2)
      MATB(16)      =  T2X - T1X
*   Y DIRECTION
      MATA(17,10)   =  -RBG2(3)
      MATA(17,12)   =  RBG2(1)
      MATA(17,16)   =  IXYT(2)
      MATA(17,17)   =  -IYYT(2)
      MATA(17,18)   =  IYZT(2)
      MATA(17,19)   =  RAG2(3)
      MATA(17,21)   =  -RAG2(1)
      MATB(17)      =  T2Y - T1Y
*   Z DIRECTION
      MATA(18,10)   =  RBG2(2)
      MATA(18,11)   =  -RBG2(1)
      MATA(18,16)   =  IXZT(2)
      MATA(18,17)   =  IYZT(2)
      MATA(18,18)   =  -IZZT(2)
      MATA(18,19)   =  -RAG2(2)
      MATA(18,20)   =  RAG2(1)
      MATB(18)      =  T2Z - T1Z

**   LINK THREE
*   SUM OF FORCES
*   X DIRECTION
530   MATA(19,19)   =  1.0D0
      MATA(19,22)   =  M3
      MATB(19)      =  0.0D0
*   Y DIRECTION
      MATA(20,20)   =  1.0D0
      MATA(20,23)   =  M3

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      MATB(20)      =  0.0D0
*    Z DIRECTION
      MATA(21,21) =  1.0D0
      MATA(21,24) =  M3
      MATB(21) = -WG3
*    EQUATIONS AT JOINT TWO
*    X DIRECTION
      MATA(22,13) = -1.0D0
      MATA(22,17) = -RAG2(3)
      MATA(22,18) =  RAG2(2)
      MATA(22,22) =  1.0D0
      MATA(22,26) =  RBG3(3)
      MATA(22,27) = -RBG3(2)
      MATB(22)      =  MIC3 - MIC4
*    Y DIRECTION
      MATA(23,14) = -1.0D0
      MATA(23,16) =  RAG2(3)
      MATA(23,18) = -RAG2(1)
      MATA(23,23) =  1.0D0
      MATA(23,25) = -RBG3(3)
      MATA(23,27) =  RBG3(1)
      MATB(23)      =  MJC3 - MJC4
*    Z DIRECTION
      MATA(24,15) = -1.0D0
      MATA(24,16) = -RAG2(2)
      MATA(24,17) =  RAG2(1)
      MATA(24,24) =  1.0D0
      MATA(24,25) =  RBG3(2)
      MATA(24,26) = -RBG3(1)
      MATB(24)      =  MKC3 - MKC4
*    SUM OF MOMENTS EQUATIONS
*    X DIRECTION
      MATA(25,20) =  RBG3(3)
      MATA(25,21) = -RBG3(2)
      MATA(25,25) = -IXXT(3)
      MATA(25,26) =  IXYT(3)
      MATA(25,27) =  IXZT(3)
      MATB(25)      = -T2X
*    Y DIRECTION
      MATA(26,19) = -RBG3(3)
      MATA(26,21) =  RBG3(1)
      MATA(26,25) =  IXYT(3)
      MATA(26,26) = -IYYT(3)
      MATA(26,27) =  IYZT(3)
      MATB(26)      = -T2Y
*    Z DIRECTION
      MATA(27,19) =  RBG3(2)
      MATA(27,20) = -RBG3(1)
      MATA(27,25) =  IXZT(3)
      MATA(27,26) =  IYZT(3)
      MATA(27,27) = -IZZT(3)
      MATB(27)      = -T2Z

***** FIRST LINK IS CONSTRAINED TO ROTATE IN Z DIRECTION *****
590 DO 600 I = 4,8
      DO 595 J = 1,27

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```

      MATA(I,J) = 0.0D0
*      MATA(J,I) = 0.0D0
595      CONTINUE
      MATB(I) = 0.0D0
600      CONTINUE
      MATA(4,4) = 1.0D0
      MATA(5,5) = 1.0D0
      MATA(6,6) = 1.0D0
      MATA(7,7) = 1.0D0
      MATA(8,8) = 1.0D0

605      DO 610 J = 1,27
      MATA(18,J) = 0.0D0
      MATA(27,J) = 0.0D0
610      CONTINUE

      MATA(9,9) = -(IZZT(1)+IZZT(2)+IZZT(3))
      MATA(18,9) = -1.0D0
      MATA(18,18) = 1.0D0
      MATB(18) = 0.0D0
      MATA(27,9) = -1.0D0
      MATA(27,27) = 1.0D0
      MATB(27) = 0.0D0

*      CALL EQUATION SOLVER PROGRAM FROM IMSL
620      CALL LEQT2F(MATA,M,N,IA,MATB,IDGT,WKAREA,IER)
      IF (IER.EQ.0) THEN
      GOTO 640
      ELSE
      WRITE (*,624) IER
624      FORMAT (I5)
      DO 635 I = 1, 27
      WRITE (*,627) I
627      FORMAT (I7)
      DO 631 J = 1, 27, 3
      WRITE (*,630) J,MATA(I,J),J+1,MATA(I,J+1),J+2,MATA(I,J+2)
630      FORMAT (I5,F13.5,I5,F13.5,I5,F13.5)
631      CONTINUE
      WRITE (*,633) I,MATB(I)
633      FORMAT (I3,F15.5)
635      CONTINUE
      CALL ENDJOB
      END IF

***      FIND LCOGX,LCOGY,LCOGZ,THETA VALUES,WX,WY,WZ
***      MODIFIED BY R.M.VERBOS

*      JOINT ZERO
640      FX0 = MATB(1)
      FY0 = MATB(2)
      FZ0 = MATB(3)

*      LINK ONE
*      SINCE LINK1 IS CONSTRAIN NOT TO MOVE ALL ACC AND VEL ARE ZERO
      AX1 = 0.0D0

```

```

        AY1 = 0.0D0
        AZ1 = 0.0D0
*660      AX1      = MATB(4)
*        VELX1    = INTGRL(0.,AX1)
*        LCOGX1   = INTGRL(X1,VELX1)
*        LCOGX(1) = LCOGX1
*        AY1      = MATB(5)
*        VELY1    = INTGRL(0.,AY1)
*        LCOGY1   = INTGRL(Y1,VELY1)
*        LCOGY(1) = LCOGY1
*        AZ1      = MATB(6)
*        VELZ1    = INTGRL(0.,AZ1)
*        LCOGZ1   = INTGRL(Z1,VELZ1)
*        LCOGZ(1) = LCOGZ1
*        WD1X     = MATB(7)
*        W1X      = INTGRL(0.,WD1X)
*        WDX(1)   = WD1X
*        W1(1)    = W1X
*        WD1Y     = MATB(8)
*        W1Y      = INTGRL(0.,WD1Y)
*        WDY(1)   = WD1Y
*        W1(2)    = W1Y
*        WD1Z     = MATB(9)
*        W1Z      = INTGRL(0.,WD1Z)
*        WDZ(1)   = WD1Z
*        W1(3)    = W1Z
****
685      THZ      = INTGRL(0.,W1Z)
        THZANG   = THZ * RADEG
        COSTHZ   = DCOS(THZ)
        SINTHZ   = DSIN(THZ)

*      IF THE 1ST LINK IS CONSTRAIN TO ROTATE IN THE Z DIRECTION ONLY
*      THE DIRECTION COSINE AND DIRECTION COSINE ANGLES ARE CONSTANT
*      AND DO NOT NEED TO BE CALCULATED
*
*      CALC DIRECTIONAL COSINES FOR LINK ONE
*
*700      DRCSX(1) = LCOGX1 / LNCOG1
*      DRCSY(1) = LCOGY1 / LNCOG1
*      DRCSZ(1) = LCOGZ1 / LNCOG1
*      AMP = DSQRT(DRCSX(1)*DRCSX(1)+DRCSY(1)*DRCSY(1)+...
*              DRCSZ(1)*DRCSZ(1))
*      DRCSX(1) = DRCSX(1)/AMP
*      DRCSY(1) = DRCSY(1)/AMP
*      DRCSZ(1) = DRCSZ(1)/AMP
*720      DRCANX(1) = DACOS(DRCSX(1)) * RADEG
*      DRCANY(1) = DACOS(DRCSY(1)) * RADEG
*      DRCANZ(1) = DACOS(DRCSZ(1)) * RADEG
*
**      JOINT LOCATION
*730      JX1 = LINKL1 * DRCSX(1)
*      JY1 = LINKL1 * DRCSY(1)
*      JZ1 = LINKL1 * DRCSZ(1)

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*      JOINT ONE
735    FX1 = MATB(10)
      FY1 = MATB(11)
      FZ1 = MATB(12)
**    LINK TWO
740    AX2      = MATB(13)
      VELX2    = INTGRL(0.,AX2)
      LCOGX2   = INTGRL(X2,VELX2)
      LCOGX(2) = LCOGX2
      AY2      = MATB(14)
      VELY2    = INTGRL(0.,AY2)
      LCOGY2   = INTGRL(Y2,VELY2)
      LCOGY(2) = LCOGY2
      AZ2      = MATB(15)
      VELZ2    = INTGRL(0.,AZ2)
      LCOGZ2   = INTGRL(Z2,VELZ2)
      LCOGZ(2) = LCOGZ2
      WD2X     = MATB(16)
      W2X      = INTGRL(0.,WD2X)
      WDX(2)   = WD2X
      W2(1)    = W2X
      WD2Y     = MATB(17)
      W2Y      = INTGRL(0.,WD2Y)
      WDY(2)   = WD2Y
      W2(2)    = W2Y
      WD2Z     = MATB(18)
      W2Z      = INTGRL(0.,WD2Z)
      WDZ(2)   = WD2Z
      W2(3)    = W2Z

*      GET THE DIRECTION COSINES FOR THE LINK TWO
      DRCSX(2) = (LCOGX2 - JX1) / LNCOG2
      DRCSY(2) = (LCOGY2 - JY1) / LNCOG2
      DRCSZ(2) = (LCOGZ2 - JZ1) / LNCOG2
790    AMP = DSQRT(DRCSX(2)*DRCSX(2)+DRCSY(2)*DRCSY(2)+...
      DRCSZ(2)*DRCSZ(2))
      DRCSX(2) = DRCSX(2)/AMP
      DRCSY(2) = DRCSY(2)/AMP
      DRCSZ(2) = DRCSZ(2)/AMP
      DRCANX(2) = DACOS(DRCSX(2)) * RADEG
      DRCANY(2) = DACOS(DRCSY(2)) * RADEG
      DRCANZ(2) = DACOS(DRCSZ(2)) * RADEG

*      JOINT LOCATION
800    JX2 = JX1 + LINKL2 * DRCSX(2)
      JY2 = JY1 + LINKL2 * DRCSY(2)
      JZ2 = JZ1 + LINKL2 * DRCSZ(2)

*      JOINT TWO
805    FX2 = MATB(19)
      FY2 = MATB(20)
      FZ2 = MATB(21)

**    LINK THREE
812    AX3      = MATB(22)
      VELX3    = INTGRL(0.,AX3)

```

```

LCOGX3      = INTGRL(X3,VELX3)
LCOGX(3)    = LCOGX3
AY3         = MATB(23)
VELY3       = INTGRL(0.,AY3)
LCOGY3      = INTGRL(Y3,VELY3)
LCOGY(3)    = LCOGY3
AZ3         = MATB(24)
VELZ3       = INTGRL(0.,AZ3)
LCOGZ3      = INTGRL(Z3,VELZ3)
LCOGZ(3)    = LCOGZ3
WD3X        = MATB(25)
W3X         = INTGRL(0.,WD3X)
WDX(3)      = WD3X
W3(1)       = W3X
WD3Y        = MATB(26)
W3Y         = INTGRL(0.,WD3Y)
WDY(3)      = WD3Y
W3(2)       = W3Y
WD3Z        = MATB(27)
W3Z         = INTGRL(0.,WD3Z)
WDZ(3)      = WD3Z
W3(3)       = W3Z

```

\* CALC DIRECTIONAL COSINES FOR LINK THREE

```

845      DRCSX(3) = (LCOGX3 - JX2) / LNCOG3
          DRCSY(3) = (LCOGY3 - JY2) / LNCOG3
          DRCSZ(3) = (LCOGZ3 - JZ2) / LNCOG3
865      AMP = DSQRT(DRCSX(3)*DRCSX(3)+DRCSY(3)*DRCSY(3)+...
          DRCSZ(3)*DRCSZ(3))
          DRCSX(3) = DRCSX(3)/AMP
          DRCSY(3) = DRCSY(3)/AMP
          DRCSZ(3) = DRCSZ(3)/AMP
          DRCANX(3) = DACOS(DRCSX(3)) * RADEG
          DRCANY(3) = DACOS(DRCSY(3)) * RADEG
          DRCANZ(3) = DACOS(DRCSZ(3)) * RADEG

```

\* TIP LOCATION

```

*875      TIPX = JX2 + LINKL3 * DRCSX(3)
*          TIPY = JY2 + LINKL3 * DRCSY(3)
*          TIPZ = JZ2 + LINKL3 * DRCSZ(3)

```

\* FIND THE ANGLE BETWEEN THE LIMKS

```

880      ANG23X = DRCANX(2) - DRCANX(3)
          ANG23Y = DRCANY(2) - DRCANY(3)
          ANG23Z = DRCANZ(2) - DRCANZ(3)
          ANG23 = ANG23X + ANG23Y + ANG23Z

```

```

*          ANG12X = DRCANX(1) - DRCANX(2)
*          ANG12Y = DRCANY(1) - DRCANY(2)
*          ANG12Z = DRCANZ(1) - DRCANZ(2)

```

\* DOUBLE PENDULUM CALCULATION

```

PA      = L3*M3P*SIN(TH2-TH3)*TH3DOT**2
PB      = SIN(TH2)*G*(M3P+M2P)
PC      = L3*M3P*COS(TH2-TH3)
PD      = L2*(M3P+M2P)

```

```

PE      = L2*SIN(TH2-TH3)*TH2DOT**2
PF      = SIN(TH3)*G
PG      = COS(TH2-TH3)*L2
PH      = L3
TH3DDT  = (PE-PF+((PG*PA+PG*PB)/PD))/((PH-(PG*PC/PD)))
TH2DDT  = (-PA - PB - PC*TH3DDT)/PD
TH3DOT  = INTGRL(TH3D0,TH3DDT)
TH2DOT  = INTGRL(TH2D0,TH2DDT)
TH3     = INTGRL(TH30,TH3DOT)
TH2     = INTGRL(TH20,TH2DOT)
TH2ANG  = TH2 * RADEG
IF(TH2ANG.GT.0.0)THEN
  ANGTH2 = 180.0 - DRCANZ(2)
ELSE
  ANGTH2 = DRCANZ(2) - 180.0
ENDIF
TH3ANG  = TH3 * RADEG
IF(TH3ANG.GT.0.0)THEN
  ANGTH3 = 180.0 - DRCANZ(3)
ELSE
  ANGTH3 = DRCANZ(3) - 180.0
ENDIF

```

```

END
STOP

```

FORTRAN

\* SUBROUTINE TO COMPUTE THE CROSS PRODUCT OF TWO VECTORS

```

SUBROUTINE CPROD(VECTA,VECTB,MI,MJ,MK)
940  IMPLICIT REAL*8 (A-Z)
      DIMENSION VECTA(3),VECTB(3)
      MI = VECTA(2) * VECTB(3) - VECTA(3) * VECTB(2)
      MJ = VECTA(3) * VECTB(1) - VECTA(1) * VECTB(3)
      MK = VECTA(1) * VECTB(2) - VECTA(2) * VECTB(1)
      RETURN
END

```



## APPENDIX C

### THREE DIMENSION VALIDATION PROGRAM

TITLE NONSINGULAR 3-D VALIDATION PROGRAM

```
*****
* THIS IS A PROGRAM WHICH VALIDATES THE NON-SINGULARITY OF A THREE *
* LINK RIGID BODY MANIPULATOR WITHOUT THE USE OF THE STANDARD ROBOT *
* TRANSFORMATION MATRICES. THE ORIGINAL PROGRAM WAS WRITTEN BY LT *
* ATINOK AND HAS BEEN GREATLY MODIFIED TO INCLUDE THREE DIMENSIONAL *
* MOTION AND GRAVITY BY LT ROBERT M. VERBOS. SEPTEMBER 1988 *
*****
```

TERMINAL

METHOD ADAMS

PRINT .02,ERROR1,ERROR2,ERROR3, ANG12, ANG23

CONTROL FINTIM =1.6, DELT = .0005, DELMAX = .1, DELPRT = .02

D DIMENSION MATA(27,27),MASS(3,2),L(3,2),RX(3,2),RY(3,2),RZ(3,2)

D DIMENSION IXX(3,2),IXZ(3,2),IXY(3,2),IYY(3,2),IYZ(3,2),IZZ(3,2)

D DIMENSION IMAT(3,3,3),NIMAT(3,3),TMAT(3,3),TMATTR(3,3),MATTMP(3,3)

D DIMENSION LIMAT(3,3,3)

FIXED IER,I,J,M,K,P,N,IA,IDGT,A,I1

ARRAY MATB(27),MATC(27),MATD(27),LCOGX(3),LCOGY(3),LCOGZ(3)

ARRAY VECTA(3),VECTB(3)

ARRAY WDX(3),WDY(3),WDZ(3),W1(3),W2(3),W3(3)

ARRAY RBG1(3),RAG1(3),RBG2(3),RAG2(3),RBG3(3)

ARRAY WKAREA(850)

ARRAY IXNT(3),IYYT(3),IZZT(3),IXYT(3),IXZT(3),IYZT(3)

ARRAY DRCANX(3),DRCANY(3),DRCANZ(3)

ARRAY DRCSX(3),DRCSY(3),DRCSZ(3),

INITIAL

\*\*\*\*\* INPUT \*\*\*\*\*

\* INPUT PARAMETER CONSTANTS

55 A = 15.0D0

P = 0.0D0

W = PI/2.0

IDGT = 6

G = 9.81D0

N = 27

M = 1

IA = 27

IER = 0

LEVELQ = 0

LEVLDQ = 0

\* INPUT JOINT LOCATIONS IN METERS

JX0 = 0.0D0

JY0 = 0.0D0

```

      JZ0 = 0.0D0
      JX1 = 0.0D0
      JY1 = 0.0D0
      JZ1 = 0.2D0
      JX2 = 0.0D0
      JY2 = 0.416D0
      JZ2 = 0.2D0
*     INPUT DISTANCE FROM CENTER OF LINK TO CENTER OF MASS
      L(1,1) = 0.05D0
      L(1,2) = 0.15D0
      L(2,1) = 0.208D0
      L(2,2) = 0.208D0
      L(3,1) = 0.2255D0
      L(3,2) = 0.2255D0
*     INPUT THE LINK LENGTHS
      LINKL1 = 0.20D0
      LINKL2 = 0.416D0
      LINKL3 = 0.451D0
*     INPUT MASS AT LINK ENDS IN KILOGRAMS
83      MASS(1,1) = 1.0D0
      MASS(1,2) = 3.0D0
      MASS(2,1) = 2.2D0
      MASS(2,2) = 2.2D0
      MASS(3,1) = 8.6D0
      MASS(3,2) = 8.6D0
*     INPUT LOCATION OF LINK CENTERS OF GRAVITY
90      LCOGX(1) = 0.0D0
      X1        = LCOGX(1)
      LCOGY(1) = 0.0D0
      Y1        = LCOGY(1)
      LCOGZ(1) = 0.10D0
      Z1        = LCOGZ(1)
      LCOGX(2) = 0.0D0
      X2        = LCOGX(2)
      LCOGY(2) = 0.208D0
      Y2        = LCOGY(2)
      LCOGZ(2) = 0.20D0
      Z2        = LCOGZ(2)
      LCOGX(3) = 0.0D0
      X3        = LCOGX(3)
      LCOGY(3) = 0.6415D0
      Y3        = LCOGY(3)
      LCOGZ(3) = 0.2D0
      Z3        = LCOGZ(3)
*     INPUT MASS OF EACH LINK IN KG
      M1 = 4.0D0
      M2 = 4.4D0
      M3 = 17.2D0
*     INPUT ACCELERATIONS OF JOINT ZERO
      AX0 = 0.0D0
      AY0 = 0.0D0
      AZ0 = 0.0D0
*     INPUT THE INITIAL DIRECTION COSINES
      DRCSX(1) = 0.0D0
      DRCSY(1) = 0.0D0
      DRCSZ(1) = 1.0D0

```

```

        DRCSX(2) = 0.0D0
        DRCSY(2) = 1.0D0
        DRCSZ(2) = 0.0D0
        DRCSX(3) = 0.0D0
        DRCSY(3) = 1.0D0
        DRCSZ(3) = 0.0D0
*      INPUT THE INITIAL DIRECTION COSINE ANGLES
        DRCANX(1) = 90.0D0
        DRCANY(1) = 90.0D0
        DRCANZ(1) = 0.0D0
        DRCANX(2) = 90.0D0
        DRCANY(2) = 0.0D0
        DRCANZ(2) = 90.0D0
        DRCANX(3) = 90.0D0
        DRCANY(3) = 0.0D0
        DRCANZ(3) = 90.0D0

*****      INITIALIZE      *****
*      OMEGA AND OMEGA DOT
139      DO 146 I = 1,3
            W1(I) = 0.0D0
            W2(I) = 0.0D0
            W3(I) = 0.0D0
            WDX(I) = 0.0D0
            WDY(I) = 0.0D0
            WDZ(I) = 0.0D0
146      CONTINUE
            W1(3) = 2.0D0
            W2(1) = 2.0D0
            W3(1) = 2.0D0
            WDY(2) = 4.0D0
            WDY(3) = 4.0D0

            THZ = 0.0D0
*      INITIALIZE MATRICES TO ZERO
        DO 180 I = 1,27
            DO 175 J = 1,27
                MATA(I,J) = 0.0D0
175      CONTINUE
                MATB(I) = 0.0D0
                MATC(I) = 0.0D0
                MATD(I) = 0.0D0
180      CONTINUE

*****      CALCULATIONS      *****
*      WEIGHTS (NEWTONS)
185      WG1 = M1*G
            WG2 = M2*G
            WG3 = M3*G
*      COMPUTE THE LENGTH FROM THE INBOARD JOINT TO COG
        LNCOG1 = DSQRT ( LCOGX(1)*LCOGX(1) + LCOGY(1)*LCOGY(1) +...
                        LCOGZ(1)*LCOGZ(1) )
        LX2 = LCOGX(2) - JX1
        LY2 = LCOGY(2) - JY1
        LZ2 = LCOGZ(2) - JZ1
        LNCOG2 = DSQRT ( LX2*LX2 + LY2*LY2 + LZ2*LZ2 )

```

```

      LX3 = LCOGX(3) - JX2
      LY3 = LCOGY(3) - JY2
      LZ3 = LCOGZ(3) - JZ2
      LNCOG3 = DSQRT ( LX3*LX3 + LY3*LY3 + LZ3*LZ3 )
*      COMPUTE FBD INERTIAS BASED ON POINT MASSES IN GLOBAL COORDINATES
190      DO 225 I = 1,3
          RX(I,1) = -L(I,1) * DRCSX(I)
          RX(I,2) =  L(I,2) * DRCSX(I)
          RY(I,1) = -L(I,1) * DRCSY(I)
          RY(I,2) =  L(I,2) * DRCSY(I)
          RZ(I,1) = -L(I,1) * DRCSZ(I)
          RZ(I,2) =  L(I,2) * DRCSZ(I)
200      IXX(I,1) = MASS(I,1) * ((RY(I,1) * RY(I,1)) + (RZ(I,1) * RZ(I,1)))
      IXX(I,2) = MASS(I,2) * ((RY(I,2) * RY(I,2)) + (RZ(I,2) * RZ(I,2)))
      IXXT(I)  = IXX(I,1) + IXX(I,2)
      IYY(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RZ(I,1) * RZ(I,1)))
      IYY(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RZ(I,2) * RZ(I,2)))
      IYYT(I)  = IYY(I,1) + IYY(I,2)
      IZZ(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RY(I,1) * RY(I,1)))
      IZZ(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RY(I,2) * RY(I,2)))
      IZZT(I)  = IZZ(I,1) + IZZ(I,2)
205      IXY(I,1) = MASS(I,1) * RX(I,1) * RY(I,1)
      IXY(I,2) = MASS(I,2) * RX(I,2) * RY(I,2)
      IXYT(I)  = IXY(I,1) + IXY(I,2)
      IXZ(I,1) = MASS(I,1) * RZ(I,1) * RX(I,1)
      IXZ(I,2) = MASS(I,2) * RZ(I,2) * RX(I,2)
      IXZT(I)  = IXZ(I,1) + IXZ(I,2)
      IYZ(I,1) = MASS(I,1) * RY(I,1) * RZ(I,1)
      IYZ(I,2) = MASS(I,2) * RY(I,2) * RZ(I,2)
      IYZT(I)  = IYZ(I,1) + IYZ(I,2)
      IF (IXXT(I) .LE. .01) THEN
          IXXT(I) = .01
      ELSE
          IXXT(I) = IXXT(I)
      END IF
      IF (IYYT(I) .LE. .01) THEN
          IYYT(I) = .01
      ELSE
          IYYT(I) = IYYT(I)
      END IF
      IF (IZZT(I) .LE. .01) THEN
          IZZT(I) = .01
      ELSE
          IZZT(I) = IZZT(I)
      END IF
      IMAT(I,1,1) = IXXT(I)
      IMAT(I,1,2) = IXYT(I)
      IMAT(I,1,3) = IXZT(I)
      IMAT(I,2,1) = -IXYT(I)
      IMAT(I,2,2) = IYYT(I)
      IMAT(I,2,3) = IYZT(I)
      IMAT(I,3,1) = -IXZT(I)
      IMAT(I,3,2) = -IYZT(I)
      IMAT(I,3,3) = IZZT(I)
225      CONTINUE
*      DUE TO LINK 1 CONSTRAINTS AND SYMMETRY LINK 1 INERTIAS ARE CONST

```

```

IXXT(1) = IMAT(1,1,1)
IXYT(1) = IMAT(1,1,2)
IXZT(1) = IMAT(1,1,3)
IYYT(1) = IMAT(1,2,2)
IYZT(1) = IMAT(1,2,3)
IZZT(1) = IMAT(1,3,3)
*   TRANSFORM THE GLOBAL INERTIAS TO LOCAL WHICH REMAIN CONSTANT
      DO 281 I = 2, 3
        TMAT(2,1) = -DCOS ( THZ )
        TMAT(2,2) = -DSIN ( THZ )
        TMAT(2,3) = 0.0D0
        TMAT(3,1) = DRCSX(I)
        TMAT(3,2) = DRCSY(I)
        TMAT(3,3) = DRCSZ(I)
        VECTA(1) = TMAT(2,1)
        VECTA(2) = TMAT(2,2)
        VECTA(3) = TMAT(2,3)
        VECTB(1) = TMAT(3,1)
        VECTB(2) = TMAT(3,2)
        VECTB(3) = TMAT(3,3)
        CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
        TMAT(1,1) = MI1
        TMAT(1,2) = MJ1
        TMAT(1,3) = MK1
        TMATTR(1,1) = TMAT(1,1)
        TMATTR(1,2) = TMAT(2,1)
        TMATTR(1,3) = TMAT(3,1)
        TMATTR(2,1) = TMAT(1,2)
        TMATTR(2,2) = TMAT(2,2)
        TMATTR(2,3) = TMAT(3,2)
        TMATTR(3,1) = TMAT(1,3)
        TMATTR(3,2) = TMAT(2,3)
        TMATTR(3,3) = TMAT(3,3)
        DO 272 I1 = 1,3
          DO 272 J = 1,3
            TEMP = 0.0D0
            DO 270 K = 1,3
              TEMP = TMAT(I1,K) * IMAT(I,K,J) + TEMP
270          CONTINUE
            MATTMP(I1,J) = TEMP
272          CONTINUE
          DO 280 I1 = 1,3
            DO 280 J = 1,3
              TEMP = 0.0D0
              DO 278 K = 1,3
                TEMP = MATTMP(I1,K) * TMATTR(K,J) + TEMP
278              CONTINUE
              LIMAT(I,I1,J) = TEMP
280              CONTINUE
281          CONTINUE

DERIVATIVE
NOSORT
287          CALL ERRSET (208,256,-1,1,1)
          CALL UERSET(LEVELQ,LEVLDQ)
*   INITIALIZE MATRICES TO ZERO

```



```

DO 297 I = 1,27
  DO 293 J = 1,27
    MATA(I,J) = 0.0D0
293    CONTINUE
    MATB(I) = 0.0D0
    MATC(I) = 0.0D0
    MATD(I) = 0.0D0
297    CONTINUE

*   VALIDATION PORTION OF PROGRAM ***
*   ENTER THE ANGULAR AND LINEAR ACCELERATIONS, AND FORCES INTO MATRIX C
*   FOR THE CONSTRAIN OF THE NEPTUNE II
    TH1L      = 2.0 * TIME + PI/2.0
    W1L       = 2.0D0
    WD1L      = 0.0D0
    TH1ANG    = TH1L * RADEG
    WDX1G     = 0.0D0
    WDY1G     = 0.0D0
    WDZ1G     = WD1L
    WX1G      = 0.0D0
    WY1G      = 0.0D0
    WZ1G      = W1L
    TH2L      = 2.0 * TIME
    W2L       = 2.0
    WD2L      = 0.0D0
    TH2ANG    = TH2L * RADEG
    WX2G      = W2L * SIN(TH1L)
    WY2G      = W2L * (- COS(TH1L) )
    WZ2G      = W1L
    VECTA(1)  = 0.0D0
    VECTA(2)  = 0.0D0
    VECTA(3)  = WZ2G
    VECTB(1)  = WX2G
    VECTB(2)  = WY2G
    VECTB(3)  = 0.0D0
    CALL CPRD (VECTA,VECTB,MI1,MJ1,MK1)
    WDX2G     = MI1
    WDY2G     = MJ1
    WDZ2G     = MK1
    TH3L      = 0.0D0
    W3L       = 0.0D0
    WD3L      = 0.0D0
    TH3ANG    = TH3L * RADEG
    WX3G      = WX2G
    WY3G      = WY2G
    WZ3G      = WZ2G
    WDX3G     = WDX2G
    WDY3G     = WDY2G
    WDZ3G     = WDZ2G
    R2X       = LNCOG2 * COS(TH2L) * COS(TH1L)
    R2Y       = LNCOG2 * COS(TH2L) * SIN(TH1L)
    R2Z       = LNCOG2 * SIN(TH2L)
    RJ2X      = LINKL2 * COS(TH2L) * COS(TH1L)
    RJ2Y      = LINKL2 * COS(TH2L) * SIN(TH1L)
    RJ2Z      = LINKL2 * SIN(TH2L)
    R3X       = RJ2X + LNCOG3 * COS(TH2L+TH3L) * COS(TH1L)

```

```

R3Y      = RJ2Y + LNCOG3 * COS(TH2L+TH3L) * SIN(TH1L)
R3Z      = RJ2Z + LNCOG3 * SIN(TH2L+TH3L)
LTIPX    = RJ2X + LINKL3 * COS(TH2L+TH3L) * COS(TH1L)
LTIPY    = RJ2Y + LINKL3 * COS(TH2L+TH3L) * SIN(TH1L)
LTIPZ    = RJ2Z + LINKL3 * SIN(TH2L+TH3L) + JZ1
*
CALCULATE THE GLOBAL ACCELERATIONS
AX1G     = 0.0D0
AY1G     = 0.0D0
AZ1G     = 0.0D0
VECTA(1) = WDX2G
VECTA(2) = WDY2G
VECTA(3) = WDZ2G
VECTB(1) = R2X
VECTB(2) = R2Y
VECTB(3) = R2Z
CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
VECTA(1) = WX2G
VECTA(2) = WY2G
VECTA(3) = WZ2G
CALL CPROD (VECTA,VECTB,MI2,MJ2,MK2)
VECTB(1) = MI2
VECTB(2) = MJ2
VECTB(3) = MK2
CALL CPROD (VECTA,VECTB,MI2,MJ2,MK2)
AX2G     = MI1 + MI2
AY2G     = MJ1 + MJ2
AZ2G     = MK1 + MK2
VECTA(1) = WDX3G
VECTA(2) = WDY3G
VECTA(3) = WDZ3G
VECTB(1) = R3X
VECTB(2) = R3Y
VECTB(3) = R3Z
CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
VECTA(1) = WX3G
VECTA(2) = WY3G
VECTA(3) = WZ3G
CALL CPROD (VECTA,VECTB,MI2,MJ2,MK2)
VECTB(1) = MI2
VECTB(2) = MJ2
VECTB(3) = MK2
CALL CPROD (VECTA,VECTB,MI2,MJ2,MK2)
AX3G     = MI1 + MI2
AY3G     = MJ1 + MJ2
AZ3G     = MK1 + MK2
*
SET MATRIX D
MATD(27) = WDZ3G
MATD(26) = WDY3G
MATD(25) = WDX3G
MATD(24) = AZ3G
MATD(23) = AY3G
MATD(22) = AX3G
MATD(21) = -M3 * AZ3G - WG3
MATD(20) = -M3 * AY3G
MATD(19) = -M3 * AX3G
MATD(18) = WDZ2G

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MATD(17) = WDY2G
MATD(16) = WDX2G
MATD(15) = AZ2G
MATD(14) = AY2G
MATD(13) = AX2G
MATD(12) = MATD(21) - M2 * AZ2G - WG2
MATD(11) = MATD(20) - M2 * AY2G
MATD(10) = MATD(19) - M2 * AX2G
MATD(9) = WDZ1G
MATD(8) = WDY1G
MATD(7) = WDX1G
MATD(6) = AZ1G
MATD(5) = AY1G
MATD(4) = AX1G
MATD(3) = MATD(12) - M1 * AZ1G - WG1
MATD(2) = MATD(11) - M1 * AY1G
MATD(1) = MATD(10) - M1 * AX1G

```

\*\*\* DIRECT DYNAMICS PORTION OF PROGRAM \*\*\*

\* INPUT JOINT EQUATIONS

\* JOINT ZERO EQUATIONS

\* W1 X (W1 X RB/G1)

RBG1(1) = JX0 - LCOGX(1)

RBG1(2) = JY0 - LCOGY(1)

RBG1(3) = JZ0 - LCOGZ(1)

VECTA(1) = W1(1)

VECTA(2) = W1(2)

VECTA(3) = W1(3)

CALL CPROD(VECTA, RBG1, MICO, MJC0, MKC0)

VECTB(1) = MICO

VECTB(2) = MJC0

VECTB(3) = MKC0

CALL CPROD(VECTA, VECTB, MICO, MJC0, MKC0)

\* W1 X (W1 X RA/G1)

RAG1(1) = JX1 - LCOGX(1)

RAG1(2) = JY1 - LCOGY(1)

RAG1(3) = JZ1 - LCOGZ(1)

VECTA(1) = W1(1)

VECTA(2) = W1(2)

VECTA(3) = W1(3)

CALL CPROD(VECTA, RAG1, MIC1, MJC1, MKC1)

VECTB(1) = MIC1

VECTB(2) = MJC1

VECTB(3) = MKC1

CALL CPROD(VECTA, VECTB, MIC1, MJC1, MKC1)

\* W2 X (W2 X RB/G2)

RBG2(1) = JX1 - LCOGX(2)

RBG2(2) = JY1 - LCOGY(2)

RBG2(3) = JZ1 - LCOGZ(2)

VECTA(1) = W2(1)

VECTA(2) = W2(2)

VECTA(3) = W2(3)

CALL CPROD(VECTA, RBG2, MIC2, MJC2, MKC2)

VECTB(1) = MIC2

VECTB(2) = MJC2

VECTB(3) = MKC2

```

      CALL CPROD (VECTA,VECTB,MIC2,MJC2,MKC2)
* W2 X (W2 X RA/G2)
      RAG2(1) = JX2 - LCOGX(2)
      RAG2(2) = JY2 - LCOGY(2)
      RAG2(3) = JZ2 - LCOGZ(2)
      VECTA(1) = W2(1)
      VECTA(2) = W2(2)
      VECTA(3) = W2(3)
      CALL CPROD (VECTA,RAG2,MIC3,MJC3,MKC3)
      VECTB(1) = MIC3
      VECTB(2) = MJC3
      VECTB(3) = MKC3
      CALL CPROD(VECTA,VECTB,MIC3,MJC3,MKC3)
* W3 X (W3 X RB/G3)
      RBG3(1) = JX2 - LCOGX(3)
      RBG3(2) = JY2 - LCOGY(3)
      RBG3(3) = JZ2 - LCOGZ(3)
      VECTA(1) = W3(1)
      VECTA(2) = W3(2)
      VECTA(3) = W3(3)
      CALL CPROD (VECTA,RBG3,MIC4,MJC4,MKC4)
      VECTB(1) = MIC4
      VECTB(2) = MJC4
      VECTB(3) = MKC4
      CALL CPROD (VECTA,VECTB,MIC4,MJC4,MKC4)
* INERTIA TRANSFORMATION
      TMAT(2,1) = -DCOS ( THZ )
      TMAT(2,2) = -DSIN ( THZ )
      TMAT(2,3) = 0.0D0
      DO 880 I = 2, 3
      TMAT(3,1) = DRCSX(I)
      TMAT(3,2) = DRCSY(I)
      TMAT(3,3) = DRCSZ(I)
      VECTA(1) = TMAT(2,1)
      VECTA(2) = TMAT(2,2)
      VECTA(3) = TMAT(2,3)
      VECTB(1) = TMAT(3,1)
      VECTB(2) = TMAT(3,2)
      VECTB(3) = TMAT(3,3)
      CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
      TMAT(1,1) = MI1
      TMAT(1,2) = MJ1
      TMAT(1,3) = MK1
      TMATTR(1,1) = TMAT(1,1)
      TMATTR(1,2) = TMAT(2,1)
      TMATTR(1,3) = TMAT(3,1)
      TMATTR(2,1) = TMAT(1,2)
      TMATTR(2,2) = TMAT(2,2)
      TMATTR(2,3) = TMAT(3,2)
      TMATTR(3,1) = TMAT(1,3)
      TMATTR(3,2) = TMAT(2,3)
      TMATTR(3,3) = TMAT(3,3)
      DO 866 I1 = 1,3
      DO 866 J = 1,3
      TEMP = 0.0D0
      DO 864 K = 1,3

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      TEMP = TMATTR(I1,K) * LIMAT(I,K,J) + TEMP
864      CONTINUE
      MATTMP(I1,J) = TEMP
866      CONTINUE
      DO 874 I1 = 1,3
      DO 874 J = 1,3
      TEMP = 0.0D0
      DO 872 K = 1,3
      TEMP = MATTMP(I1,K) * TMAT(K,J) + TEMP
872      CONTINUE
      NIMAT(I1,J) = TEMP
874      CONTINUE
      IXXT(I) = NIMAT(1,1)
      IXYT(I) = NIMAT(1,2)
      IXZT(I) = NIMAT(1,3)
      IYYT(I) = NIMAT(2,2)
      IYZT(I) = NIMAT(2,3)
      IZZT(I) = NIMAT(3,3)
880      CONTINUE

*      ENTER CONSTANS INTO MATRIX A
**      LINK ONE
*      SUM OF FORCES
*      X DIRECTION
887      MATA(1,1) = 1.0D0
      MATA(1,4) = M1
      MATA(1,10) = -1.0D0
*      Y DIRECTION
      MATA(2,2) = 1.0D0
      MATA(2,5) = M1
      MATA(2,11) = -1.0D0
*      Z DIRECTION
      MATA(3,3) = 1.0D0
      MATA(3,6) = M1
      MATA(3,12) = -1.0D0
*      EQUATIONS AT JOINT ZERO
*      X DIRECTION
      MATA(4,4) = 1.0D0
      MATA(4,8) = RBG1(3)
      MATA(4,9) = -RBG1(2)
*      Y DIRECTION
      MATA(5,5) = 1.0D0
      MATA(5,7) = -RBG1(3)
      MATA(5,9) = RBG1(1)
*      Z DIRECTION
      MATA(6,6) = 1.0D0
      MATA(6,7) = RBG1(2)
      MATA(6,8) = -RBG1(1)
*      SUM OF MOMENTS EQUATIONS
*      X DIRECTION
      MATA(7,2) = RBG1(3)
      MATA(7,3) = -RBG1(2)
      MATA(7,7) = -IXXT(1)
      MATA(7,8) = IXYT(1)
      MATA(7,9) = IXZT(1)
      MATA(7,11) = -RAG1(3)

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      MATA(7,12) = RAG1(2)
*   Y DIRECTION
      MATA(8,1) = -RBG1(3)
      MATA(8,3) = RBG1(1)
      MATA(8,7) = IXYT(1)
      MATA(8,8) = -IYYT(1)
      MATA(8,9) = IYZT(1)
      MATA(8,10) = RAG1(3)
      MATA(8,12) = -RAG1(1)
*   Z DIRECTION
      MATA(9,1) = RBG1(2)
      MATA(9,2) = -RBG1(1)
      MATA(9,7) = IXZT(1)
      MATA(9,8) = IYZT(1)
      MATA(9,9) = -IZZT(1)
      MATA(9,10) = -RAG1(2)
      MATA(9,11) = RAG1(1)
**  LINK TWO
*   SUM OF FORCES
*   X DIRECTION
939   MATA(10,10) = 1.0D0
      MATA(10,13) = M2
      MATA(10,19) = -1.0D0
*   Y DIRECTION
      MATA(11,11) = 1.0D0
      MATA(11,14) = M2
      MATA(11,20) = -1.0D0
*   Z DIRECTION
      MATA(12,12) = 1.0D0
      MATA(12,15) = M2
      MATA(12,21) = -1.0D0
*   EQUATIONS AT JOINT ONE
*   X DIRECTION
      MATA(13,4) = -1.0D0
      MATA(13,8) = -RAG1(3)
      MATA(13,9) = RAG1(2)
      MATA(13,13) = 1.0D0
      MATA(13,17) = RBG2(3)
      MATA(13,18) = -RBG2(2)
*   Y DIRECTION
      MATA(14,5) = -1.0D0
      MATA(14,7) = RAG1(3)
      MATA(14,9) = -RAG1(1)
      MATA(14,14) = 1.0D0
      MATA(14,16) = -RBG2(3)
      MATA(14,18) = RBG2(1)
*   Z DIRECTION
      MATA(15,6) = -1.0D0
      MATA(15,7) = -RAG1(2)
      MATA(15,8) = RAG1(1)
      MATA(15,15) = 1.0D0
      MATA(15,16) = RBG2(2)
      MATA(15,17) = -RBG2(1)
*   SUM OF MOMENTS EQUATIONS
*   X DIRECTION
      MATA(16,11) = RBG2(3)

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      MATA(16,12) = -RBG2(2)
      MATA(16,16) = -IXXT(2)
      MATA(16,17) =  IXYT(2)
      MATA(16,18) =  IXZT(2)
      MATA(16,20) = -RAG2(3)
      MATA(16,21) =  RAG2(2)
*    Y DIRECTION
      MATA(17,10) = -RBG2(3)
      MATA(17,12) =  RBG2(1)
      MATA(17,16) =  IXYT(2)
      MATA(17,17) = -IYYT(2)
      MATA(17,18) =  IYZT(2)
      MATA(17,19) =  RAG2(3)
      MATA(17,21) = -RAG2(1)
*    Z DIRECTION
      MATA(18,10) =  RBG2(2)
      MATA(18,11) = -RBG2(1)
      MATA(18,16) =  IXZT(2)
      MATA(18,17) =  IYZT(2)
      MATA(18,18) = -IZZT(2)
      MATA(18,19) = -RAG2(2)
      MATA(18,20) =  RAG2(1)
**   LINK THREE
*    SUM OF FORCES
*    X DIRECTION
1000  MATA(19,19) =  1.0D0
      MATA(19,22) =  M3
*    Y DIRECTION
      MATA(20,20) =  1.0D0
      MATA(20,23) =  M3
*    Z DIRECTION
      MATA(21,21) =  1.0D0
      MATA(21,24) =  M3
*    EQUATIONS AT JOINT TWO
*    X DIRECTION
      MATA(22,13) = -1.0D0
      MATA(22,17) = -RAG2(3)
      MATA(22,18) =  RAG2(2)
      MATA(22,22) =  1.0D0
      MATA(22,26) =  RBG3(3)
      MATA(22,27) = -RBG3(2)
*    Y DIRECTION
      MATA(23,14) = -1.0D0
      MATA(23,16) =  RAG2(3)
      MATA(23,18) = -RAG2(1)
      MATA(23,23) =  1.0D0
      MATA(23,25) = -RBG3(3)
      MATA(23,27) =  RBG3(1)
*    Z DIRECTION
      MATA(24,15) = -1.0D0
      MATA(24,16) = -RAG2(2)
      MATA(24,17) =  RAG2(1)
      MATA(24,24) =  1.0D0
      MATA(24,25) =  RBG3(2)

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      MATA(24,26) = -RBG3(1)
*   SUM OF MOMENTS EQUATIONS
*   X DIRECTION
      MATA(25,20) = RBG3(3)
      MATA(25,21) = -RBG3(2)
      MATA(25,25) = -IXXT(3)
      MATA(25,26) = IXYT(3)
      MATA(25,27) = IXZT(3)
*   Y DIRECTION
      MATA(26,19) = -RBG3(3)
      MATA(26,21) = RBG3(1)
      MATA(26,25) = IXYT(3)
      MATA(26,26) = -IYYT(3)
      MATA(26,27) = IYZT(3)
*   Z DIRECTION
      MATA(27,19) = RBG3(2)
      MATA(27,20) = -RBG3(1)
      MATA(27,25) = IXZT(3)
      MATA(27,26) = IYZT(3)
      MATA(27,27) = -IZZT(3)
***** FIRST LINK IS CONSTRAINED TO ROTATE IN Z DIRECTION *****
1051  DO 1056 I = 4,8
      DO 1055 J = 1,27
          MATA(I,J) = 0.0D0
*          MATA(J,I) = 0.0D0
1055  CONTINUE
1056  CONTINUE
      MATA(4,4) = 1.0D0
      MATA(5,5) = 1.0D0
      MATA(6,6) = 1.0D0
      MATA(7,7) = 1.0D0
      MATA(8,8) = 1.0D0
      DO 1065 J = 1,27
          MATA(18,J) = 0.0D0
          MATA(27,J) = 0.0D0
1065  CONTINUE
      IZZT(2) = IZZT(2) + M2 * (DSQRT(LCOGX(2)**2+LCOGY(2)**2))**2
      IZZT(3) = IZZT(3) + M3 * (DSQRT(LCOGX(3)**2+LCOGY(3)**2))**2
      MATA(9,9) = -(IZZT(1)+IZZT(2)+IZZT(3))
      MATA(18,9) = -1.0D0
      MATA(18,18) = 1.0D0
      MATA(27,9) = -1.0D0
      MATA(27,27) = 1.0D0
*   MULTIPLY MATA * MATD TO OBTAIN TORQUES
      DO 758 J = 1,27
          SUM1 = 0.0D0
          DO 755 K = 1,27
              SUM1 = SUM1 + MATA(J,K) * MATD(K)
755  CONTINUE
      MATC(J) = SUM1
758  CONTINUE
      T2Z = -MATC(27)
      T2Y = -MATC(26)
      T2X = -MATC(25)
      T1Z = T2Z - MATC(18)
      T1Y = T2Y - MATC(17)

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      T1X = T2X - MATC(16)
      TOZ = T1Z - MATC(9)
      TOY = T1Y - MATC(8)
      TOX = T1X - MATC(7)
*      ENTER KNOWNNS INTO MATB
1077    MATB(1) = 0.0D0
      MATB(2) = 0.0D0
      MATB(3) = -WG1
      MATB(4) = AX0 - MICO
      MATB(5) = AYO - MJCO
      MATB(6) = AZO - MKCO
      MATB(7) = T1X - TOX
      MATB(8) = T1Y - TOY
      MATB(9) = T1Z - TOZ
      MATB(10) = 0.0D0
      MATB(11) = 0.0D0
      MATB(12) = -WG2
      MATB(13) = MIC1 - MIC2
      MATB(14) = MJC1 - MJC2
      MATB(15) = MKC1 - MKC2
      MATB(16) = T2X - T1X
      MATB(17) = T2Y - T1Y
      MATB(18) = T2Z - T1Z
      MATB(19) = 0.0D0
      MATB(20) = 0.0D0
      MATB(21) = -WG3
      MATB(22) = MIC3 - MIC4
      MATB(23) = MJC3 - MJC4
      MATB(24) = MKC3 - MKC4
      MATB(25) = -T2X
      MATB(26) = -T2Y
      MATB(27) = -T2Z
***** FIRST LINK IS CONSTRAINED TO ROTATE IN Z DIRECTION *****
      DO 1107 I = 4,8
        MATB(I) = 0.0D0
1107    CONTINUE
        MATB(18) = 0.0D0
        MATB(27) = 0.0D0

*      CALL EQUATION SOLVER PROGRAM FROM IMSL
1112    CALL LEQT2F(MATA,M,N,IA,MATB,IDGT,WKAREA,IER)
      IF (IER.EQ.0) THEN
        GOTO 1133
      ELSE
1117    WRITE (*,1117) IER
        FORMAT (I5)
        DO 1127 I = 1, 27
          WRITE (*,1120) I
1120    FORMAT (I7)
          DO 1124 J = 1, 27, 3
            WRITE (*,1123) J,MATA(I,J),J+1,MATA(I,J+1),J+2,MATA(I,J+2)
1123    FORMAT (I5,F13.5,I5,F13.5,I5,F13.5)
1124    CONTINUE
          WRITE (*,1126) I,MATB(I)
1126    FORMAT (I3,F15.5)
1127    CONTINUE

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        CALL ENDJOB
        END IF
***    FIND LCOGX, LCOGY, LCOGZ, THETA VALUES, WX, WY, WZ
*      JOINT ZERO
1133    FX0 = MATB(1)
        FY0 = MATB(2)
        FZ0 = MATB(3)
*      LINK ONE
*      SINCE LINK1 IS CONSTRAIN TO ROTATE AROUND THE Z ONLY
        AX1 = 0.0D0
        AY1 = 0.0D0
        AZ1 = 0.0D0
*141    AX1      = MATB(4)
*      VELX1     = INTGRL(0.,AX1)
*      LCOGX1    = INTGRL(X1,VELX1)
*      LCOGX(1)  = LCOGX1
*      AY1       = MATB(5)
*      VELY1     = INTGRL(0.,AY1)
*      LCOGY1    = INTGRL(Y1,VELY1)
*      LCOGY(1)  = LCOGY1
*      AZ1       = MATB(6)
*      VELZ1     = INTGRL(0.,AZ1)
*      LCOGZ1    = INTGRL(Z1,VELZ1)
*      LCOGZ(1)  = LCOGZ1
        WD1X     = MATB(7)
        W1X      = INTGRL(0.,WD1X)
        WDX(1)   = WD1X
        W1(1)    = W1X
        WD1Y     = MATB(8)
        W1Y      = INTGRL(0.,WD1Y)
        WDY(1)   = WD1Y
        W1(2)    = W1Y
        WD1Z     = MATB(9)
        W1Z      = INTGRL(2.,WD1Z)
        WDZ(1)   = WD1Z
        W1(3)    = W1Z
1166    THZ      = INTGRL(0.,W1Z)
        THZANG   = THZ * RADEG
        COSTHZ   = DCOS(THZ)
        SINTHZ   = DSIN(THZ)
*      IF THE 1ST LINK IS CONSTRAIN TO ROTATE IN THE Z DIRECTION ONLY
*      THE DIRECTION COSINE AND DIRECTION COSINE ANGLES ARE CONSTANT
*      AND DO NOT NEED TO BE CALCULATED
*      JOINT ONE
1174    FX1 = MATB(10)
        FY1 = MATB(11)
        FZ1 = MATB(12)
**      LINK TWO
1178    AX2      = MATB(13)
        VELX2     = INTGRL(-0.416,AX2)
        LCOGX2    = INTGRL(X2,VELX2)
        LCOGX(2)  = LCOGX2
        AY2       = MATB(14)
        VELY2     = INTGRL(0.,AY2)
        LCOGY2    = INTGRL(Y2,VELY2)
        LCOGY(2)  = LCOGY2

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    AZ2      = MATB(15)
    VELZ2    = INTGRL(0.416,AZ2)
    LCOGZ2   = INTGRL(Z2,VELZ2)
    LCOGZ(2) = LCOGZ2
    WD2X     = MATB(16)
    W2X      = INTGRL(2.,WD2X)
    WDX(2)   = WD2X
    W2(1)    = W2X
    WD2Y     = MATB(17)
    W2Y      = INTGRL(0.,WD2Y)
    WDY(2)   = WD2Y
    W2(2)    = W2Y
    WD2Z     = MATB(18)
    W2Z      = INTGRL(2.,WD2Z)
    W2(3)    = W2Z
    WDZ(2)   = WD2Z
*   GET THE DIRECTION COSINES FOR THE LINK TWO
    DRCSX(2) = (LCOGX2 - JX1) / LNCOG2
    DRCSY(2) = (LCOGY2 - JY1) / LNCOG2
    DRCSZ(2) = (LCOGZ2 - JZ1) / LNCOG2
    AMP = DSQRT(DRCSX(2)*DRCSX(2)+DRCSY(2)*DRCSY(2)+...
              DRCSZ(2)*DRCSZ(2))
    DRCSX(2) = DRCSX(2)/AMP
    DRCSY(2) = DRCSY(2)/AMP
    DRCSZ(2) = DRCSZ(2)/AMP
    DRCANX(2) = DACOS(DRCSX(2)) * RADEG
    DRCANY(2) = DACOS(DRCSY(2)) * RADEG
    DRCANZ(2) = DACOS(DRCSZ(2)) * RADEG
*   JOINT LOCATION
1215   JX2 = JX1 + LINKL2 * DRCSX(2)
       JY2 = JY1 + LINKL2 * DRCSY(2)
       JZ2 = JZ1 + LINKL2 * DRCSZ(2)

*   JOINT TWO
1220   FX2 = MATB(19)
       FY2 = MATB(20)
       FZ2 = MATB(21)
**   LINK THREE
1224   AX3      = MATB(22)
       VELX3    = INTGRL(-1.282,AX3)
       LCOGX3   = INTGRL(X3,VELX3)
       LCOGX(3) = LCOGX3
       AY3      = MATB(23)
       VELY3    = INTGRL(0.,AY3)
       LCOGY3   = INTGRL(Y3,VELY3)
       LCOGY(3) = LCOGY3
       AZ3      = MATB(24)
       VELZ3    = INTGRL(1.282,AZ3)
       LCOGZ3   = INTGRL(Z3,VELZ3)
       LCOGZ(3) = LCOGZ3
       WD3X     = MATB(25)
       W3X      = INTGRL(2.,WD3X)
       WDX(3)   = WD3X
       W3(1)    = W3X
       WD3Y     = MATB(26)
       W3Y      = INTGRL(0.,WD3Y)

```

```

WDY(3)      = WD3Y
W3(2)       = W3Y
WD3Z        = MATB(27)
W3Z         = INTGRL(2.,WD3Z)
WDZ(3)      = WD3Z
W3(3)       = W3Z
*  CALC DIRECTIONAL COSINES FOR LINK THREE
1249  DRCSX(3) = (LCOGX3 - JX2) / LNCOG3
      DRCSY(3) = (LCOGY3 - JY2) / LNCOG3
      DRCSZ(3) = (LCOGZ3 - JZ2) / LNCOG3
      AMP = DSQRT(DRCSX(3)*DRCSX(3)+DRCSY(3)*DRCSY(3)+...
                DRCSZ(3)*DRCSZ(3))
      DRCSX(3) = DRCSX(3)/AMP
      DRCSY(3) = DRCSY(3)/AMP
      DRCSZ(3) = DRCSZ(3)/AMP
      DRCANX(3) = DACOS(DRCSX(3)) * RADEG
      DRCANY(3) = DACOS(DRCSY(3)) * RADEG
      DRCANZ(3) = DACOS(DRCSZ(3)) * RADEG
*  TIP LOCATION
1261  TIPX = JX2 + LINKL3 * DRCSX(3)
      TIPY = JY2 + LINKL3 * DRCSY(3)
      TIPZ = JZ2 + LINKL3 * DRCSZ(3)
*  FIND THE ANGLE BETWEEN THE LIMKS
1265  ANG23X = DRCANX(2) - DRCANX(3)
      ANG23Y = DRCANY(2) - DRCANY(3)
      ANG23Z = DRCANZ(2) - DRCANZ(3)
      ANG12X = DRCANX(1) - DRCANX(2)
      ANG12Y = DRCANY(1) - DRCANY(2)
      ANG12Z = DRCANZ(1) - DRCANZ(2)
      ANG12  = DRCANZ(2) - DRCANZ(1)
      ANG23  = ANG23X + ANG23Y + ANG23Z

*  CLACULATE THE ERROR BETWEEN KNOWN TIP POSITION AND CALCULATED
      ERROR1 = ABS ( LTIPX - TIPX ) / 0.867 * 100.0
      ERROR2 = ABS ( LTIPY - TIPY ) / 0.867 * 100.0
      ERROR3 = ABS ( LTIPZ - TIPZ ) / 0.867 * 100.0

END
STOP

FORTRAN
*  SUBROUTINE TO COMPUTE THE CROSS PRODUCT OF TWO VECTORS

      SUBROUTINE CPROD(VECTA,VECTB,MI,MJ,MK)
1284  IMPLICIT REAL*8 (A-Z)
      DIMENSION VECTA(3),VECTB(3)
      MI = VECTA(2) * VECTB(3) - VECTA(3) * VECTB(2)
      MJ = VECTA(3) * VECTB(1) - VECTA(1) * VECTB(3)
      MK = VECTA(1) * VECTB(2) - VECTA(2) * VECTB(1)
      RETURN
      END

```

## APPENDIX D

### ROBOT VALIDATION PROGRAM

TITLE ROBOT TORQUE VALIDATION PROGRAM

```
*****
* THIS IS A PROGRAM THAT USES ACTUAL DATA FROM THE NEPTUNE II ROBOT TO *
* OBTAIN TORQUES TO INPUT INTO THE NONSINGULAR SIMULATION OF A 3 LINK *
* RIGID MANIPULATOR. THE ORIGINAL PROGRAM WAS WRITTEN BY LT ALTINOK *
* AND HAS BEEN GREATLY MODIFIED TO INCLUDE 3 D MOTION AND GRAVITATIONAL*
* EFFECTS BY LT R. M. VERBOS USN SEPTEMBER 1988 *
*****
```

TERMINAL

```
METHOD RKSFX
PRINT .10,DRCANZ(3)
CONTROL FINTIM =7.0, DELT = .01, DELMAX = .1, DELPRT = .10
SAVE .10, DRCANZ(3)
GRAPH (DE=TEK618) TIME,DRCANZ(3)
D DIMENSION MATA(27,27),MASS(3,2),L(3,2),RX(3,2),RY(3,2),RZ(3,2)
D DIMENSION IXX(3,2),IXZ(3,2),IXY(3,2),IYY(3,2),IYZ(3,2),IZZ(3,2)
D DIMENSION IMAT(3,3,3),NIMAT(3,3),TMAT(3,3),TMATTR(3,3),MATTMP(3,3)
D DIMENSION LIMAT(3,3,3)
FIXED IER,I,J,M,K,P,N,IA,IDGT,A,I1
ARRAY MATB(27),LCOGX(3),LCOGY(3),LCOGZ(3)
ARRAY VECTAO(3),VECTBO(3),VECTA1(3),VECTB1(3),VECTA2(3),VECTB2(3)
ARRAY VECTA(3),VECTB(3)
ARRAY WDX(3),WDY(3),WDZ(3),W1(3),W2(3),W3(3)
ARRAY RBG1(3),RAG1(3),RBG2(3),RAG2(3),RBG3(3)
ARRAY WKAREA(850)
ARRAY IXXT(3),IYYT(3),IZZT(3),IXYT(3),IXZT(3),IYZT(3)
ARRAY DRCANX(3),DRCANY(3),DRCANZ(3)
ARRAY DRCSX(3),DRCSY(3),DRCSZ(3)
```

INITIAL

```
***** INPUT *****
* INPUT PARAMETER CONSTANTS
    IDGT = 6
    G = 9.81D0
    N = 27
    M = 1
    IA = 27
    IER = 0
    LEVELQ = 0
    LEVLDQ = 0

* INPUT DISTANCE FROM COG OF LINK TO CENTER OF MASS
* FOR EACH LINK ENDS
```

```

        L(1,1) = 0.1953D0
        L(1,2) = 0.0651D0
        L(2,1) = 0.208D0
        L(2,2) = 0.208D0
        L(3,1) = 0.1746D0
*       L(3,2) = 0.1746D0

* INPUT THE LINK LENGTHS OF THE ROBOT
        LINKL1 = 0.2604D0
        LINKL2 = 0.416D0
        LINKL3 = 0.3498D0

* INPUT JOINT LOCATIONS IN METERS
        JX0 = 0.0D0
        JY0 = 0.0D0
        JZ0 = 0.0D0
        JX1 = 0.0D0
        JY1 = 0.0D0
        JZ1 = 0.2604D0
        JX2 = 0.0D0
        JY2 = 0.416D0
        JZ2 = 0.2604D0

* INPUT TORQUE CONSTANTS
        TOX      = 0.0D0
        TOY      = 0.0D0
        TOZ      = 0.0D0
        T1X      = 0.0D0
        T1Y      = 0.0D0
        T1Z      = 0.0D0
        T2X      = 0.0D0
        T2Y      = 0.0D0
        T2Z      = 0.0D0
        TG1      = 0.0D0
        TG2      = 0.0D0
        T1FNC    = 0.0D0
        T2FNC    = 0.0D0

* INPUT MASS AT LINK ENDS IN KILOGRAMS
110    MASS(1,1) = 2.273D0
        MASS(1,2) = 6.818D0
        MASS(2,1) = 0.455D0
        MASS(2,2) = 0.455D0
        MASS(3,1) = 5.909D0
        MASS(3,2) = 5.909D0

* INPUT LOCATION OF LINK CENTERS OF GRAVITY
120    LCOGX(1) = 0.0D0
        X1      = LCOGX(1)
        LCOGY(1) = 0.0D0
        Y1      = LCOGY(1)
        LCOGZ(1) = 0.10D0
        Z1      = LCOGZ(1)
        LCOGX(2) = 0.0D0
        X2      = LCOGX(2)
        LCOGY(2) = 0.208D0

```

```

Y2      = LCOGY(2)
LCOGZ(2) = 0.2604D0
Z2      = LCOGZ(2)
LCOGX(3) = 0.0D0
X3      = LCOGX(3)
LCOGY(3) = 0.4159D0
Y3      = LCOGY(3)
LCOGZ(3) = 0.0858D0
Z3      = LCOGZ(3)

```

```

*      INPUT MASS OF EACH LINK IN KG

```

```

M1 = 9.091D0
M2 = 0.910D0
M3 = 11.818D0

```

```

*      INPUT ACCELERATIONS OF JOINT ZERO

```

```

AX0 = 0.0D0
AY0 = 0.0D0
AZ0 = 0.0D0

```

```

*      INPUT THE INITIAL DIRECTION COSINES

```

```

DRCSX(1) = 0.0D0
DRCSY(1) = 0.0D0
DRCSZ(1) = 1.0D0
DRCSX(2) = 0.0D0
DRCSY(2) = 1.0D0
DRCSZ(2) = 0.0D0
DRCSX(3) = 0.0D0
DRCSY(3) = 0.0D0
DRCSZ(3) = -1.0D0

```

```

*      INPUT THE INITIAL DIRECTION COSINE ANGLES

```

```

DRCANX(1) = 90.0D0
DRCANY(1) = 90.0D0
DRCANZ(1) = 0.0D0
DRCANX(2) = 90.0D0
DRCANY(2) = 0.0D0
DRCANZ(2) = 90.0D0
DRCANX(3) = 90.0D0
DRCANY(3) = 90.0D0
DRCANZ(3) = 180.0D0

```

```

***** INITIALIZE *****

```

```

*      OMEGA AND OMEGA DOT

```

```

160      DO 170 I = 1,3
           W1(I)      = 0.0D0
           W2(I)      = 0.0D0
           W3(I)      = 0.0D0
           WDX(I)     = 0.0D0
           WDY(I)     = 0.0D0
           WDZ(I)     = 0.0D0

```

```

170      CONTINUE
           W3(1) = 0.158D0
           W3IC = W3(1)

```

```

THZ = 0.0D0

```



```

*      INITIALIZE MATRIX A AND B TO ZERO
          DO 180 I = 1,27
              DO 175 J = 1,27
                  MATA(I,J) = 0.0D0
175          CONTINUE
              MATB(I) = 0.0D0
180          CONTINUE

*      CONSTANT TO CONVERT STRIP DATA TO DELTA P
          RDVTOV = 0.2D0
          RAREA = 0.00312D0
          RLA = 0.0597/2.0
          RVTOP = 10.0D0
          RLBTON = 4.448D0
          RINTOM = 39.37D0
          RCNFAC = RDVTOV*RVTOP*RAREA*RLA*RLBTON*RINTOM**2

*****      CALCULATIONS      *****

*      WEIGHTS (NEWTONS)
185          WG1 = M1*G
              WG2 = M2*G
              WG3 = M3*G

*      COMPUTE THE LENGTH FROM THE INBOARD JOINT TO COG
          LNCOG1 = DSQRT ( LCOGX(1)*LCOGX(1) + LCOGY(1)*LCOGY(1) +...
                          LCOGZ(1)*LCOGZ(1) )
          LX2 = LCOGX(2) - JX1
          LY2 = LCOGY(2) - JY1
          LZ2 = LCOGZ(2) - JZ1
          LNCOG2 = DSQRT ( LX2*LX2 + LY2*LY2 + LZ2*LZ2 )
          LX3 = LCOGX(3) - JX2
          LY3 = LCOGY(3) - JY2
          LZ3 = LCOGZ(3) - JZ2
          LNCOG3 = DSQRT ( LX3*LX3 + LY3*LY3 + LZ3*LZ3 )
          V3IC = LNCOG3 * W3(1)

*      L(3,2)
          L(3,2) = (13.5 * RCNFAC) / ( MASS(3,2) * G) - LNCOG3

*      COMPUTE INITIAL INERTIAS BASED ON POINT MASSES
*      IN GLOBAL COORDINATES
190          DO 225 I = 1,3
              RX(I,1) = -L(I,1) * DRCSX(I)
              RX(I,2) = L(I,2) * DRCSX(I)
              RY(I,1) = -L(I,1) * DRCSY(I)
              RY(I,2) = L(I,2) * DRCSY(I)
              RZ(I,1) = -L(I,1) * DRCSZ(I)
              RZ(I,2) = L(I,2) * DRCSZ(I)
200          IXX(I,1) = MASS(I,1) * ((RY(I,1) * RY(I,1)) + (RZ(I,1) * RZ(I,1)))
              IXX(I,2) = MASS(I,2) * ((RY(I,2) * RY(I,2)) + (RZ(I,2) * RZ(I,2)))
              IXXT(I) = IXX(I,1) + IXX(I,2)
              IYY(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RZ(I,1) * RZ(I,1)))
              IYY(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RZ(I,2) * RZ(I,2)))
              IYYT(I) = IYY(I,1) + IYY(I,2)

```

```

      IZZ(I,1) = MASS(I,1) * ((RX(I,1) * RX(I,1)) + (RY(I,1) * RY(I,1)))
      IZZ(I,2) = MASS(I,2) * ((RX(I,2) * RX(I,2)) + (RY(I,2) * RY(I,2)))
      IZZT(I)  = IZZ(I,1) + IZZ(I,2)
205  IXY(I,1) = MASS(I,1) * RX(I,1) * RY(I,1)
      IXY(I,2) = MASS(I,2) * RX(I,2) * RY(I,2)
      IXYT(I)  = IXY(I,1) + IXY(I,2)
      IXZ(I,1) = MASS(I,1) * RZ(I,1) * RX(I,1)
      IXZ(I,2) = MASS(I,2) * RZ(I,2) * RX(I,2)
      IXZT(I)  = IXZ(I,1) + IXZ(I,2)
      IYZ(I,1) = MASS(I,1) * RY(I,1) * RZ(I,1)
      IYZ(I,2) = MASS(I,2) * RY(I,2) * RZ(I,2)
      IYZT(I)  = IYZ(I,1) + IYZ(I,2)
      IF (IXXT(I) .LE. .10) THEN
        IXXT(I) = .10
      ELSE
        IXXT(I) = IXXT(I)
      END IF
      IF (IYYT(I) .LE. .01) THEN
        IYYT(I) = .01
      ELSE
        IYYT(I) = IYYT(I)
      END IF
      IF (IZZT(I) .LE. .01) THEN
        IZZT(I) = .01
      ELSE
        IZZT(I) = IZZT(I)
      END IF
      IMAT(I,1,1) = IXXT(I)
      IMAT(I,1,2) = IXYT(I)
      IMAT(I,1,3) = IXZT(I)
      IMAT(I,2,1) = -IXYT(I)
      IMAT(I,2,2) = IYYT(I)
      IMAT(I,2,3) = IYZT(I)
      IMAT(I,3,1) = -IXZT(I)
      IMAT(I,3,2) = -IYZT(I)
      IMAT(I,3,3) = IZZT(I)
225  CONTINUE

*      DUE TO LINK 1 CONSTRAINTS LINK 1 INERTIAS ARE CONSTANT
      IXXT(1) = IMAT(1,1,1)
      IXYT(1) = IMAT(1,1,2)
      IXZT(1) = IMAT(1,1,3)
      IYYT(1) = IMAT(1,2,2)
      IYZT(1) = IMAT(1,2,3)
      IZZT(1) = IMAT(1,3,3)

```

\* TRANSFORM THE INITIAL INERTIAS TO LOCAL COORDINATED

```

      DO 9 I = 2, 3
        TMAT(2,1) = -DCOS ( THZ )
        TMAT(2,2) = -DSIN ( THZ )
        TMAT(2,3) = 0.0D0
        TMAT(3,1) = DRCSX(I)
        TMAT(3,2) = DRCSY(I)
        TMAT(3,3) = DRCSZ(I)

```

```

    VECTA(1) = TMAT(2,1)
    VECTA(2) = TMAT(2,2)
    VECTA(3) = TMAT(2,3)
    VECTB(1) = TMAT(3,1)
    VECTB(2) = TMAT(3,2)
    VECTB(3) = TMAT(3,3)
    CALL CPRCD (VECTA,VECTB,MI1,MJ1,MK1)
    TMAT(1,1) = MI1
    TMAT(1,2) = MJ1
    TMAT(1,3) = MK1
    TMATTR(1,1) = TMAT(1,1)
    TMATTR(1,2) = TMAT(2,1)
    TMATTR(1,3) = TMAT(3,1)
    TMATTR(2,1) = TMAT(1,2)
    TMATTR(2,2) = TMAT(2,2)
    TMATTR(2,3) = TMAT(3,2)
    TMATTR(3,1) = TMAT(1,3)
    TMATTR(3,2) = TMAT(2,3)
    TMATTR(3,3) = TMAT(3,3)

    DO 5 I1 = 1,3
    DO 5 J = 1,3
        TEMP = 0.0D0
        DO 1 K = 1,3
            TEMP = TMAT(I1,K) * IMAT(I,K,J) + TEMP
1        CONTINUE
        MATTMP(I1,J) = TEMP
5        CONTINUE

    DO 8 I1 = 1,3
    DO 8 J = 1,3
        TEMP = 0.0D0
        DO 7 K = 1,3
            TEMP = MATTMP(I1,K) * TMATTR(K,J) + TEMP
7        CONTINUE
        LIMAT(I,I1,J) = TEMP
8        CONTINUE
9        CONTINUE

DERIVATIVE
NOSORT
230      CALL ERRSET (208,256,-1,1,1)
        CALL UERSET(LEVELQ,LEVLDQ)

*      INITIALIZE MATRIX A AND B TO ZERO
        DO 240 I = 1,27
            DO 235 J = 1,27
                MATA(I,J) = 0.0D0
235            CONTINUE
            MATB(I) = 0.0D0
240        CONTINUE

*      INPUT JOINT EQUATIONS

```

```

*      JOINT ZERO EQUATIONS
*      W1 X (W1 X RB/G1)
          RBG1(1) = JX0 - LCOGX(1)
          RBG1(2) = JY0 - LCOGY(1)
          RBG1(3) = JZ0 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD(VECTA0,RBG1,MICO,MJCO,MKCO)
          VECTB0(1) = MICO
          VECTB0(2) = MJCO
          VECTB0(3) = MKCO
          CALL CPROD(VECTA0,VECTB0,MICO,MJCO,MKCO)

*      INPUT JOINT EQUATIONS
*      JOINT ZERO EQUATIONS
*      W1 X (W1 X RB/G1)
          RBG1(1) = JX0 - LCOGX(1)
          RBG1(2) = JY0 - LCOGY(1)
          RBG1(3) = JZ0 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD(VECTA,RBG1,MICO,MJCO,MKCO)
          VECTB(1) = MICO
          VECTB(2) = MJCO
          VECTB(3) = MKCO
          CALL CPROD(VECTA,VECTB,MICO,MJCO,MKCO)

*      W1 X (W1 X RA/G1)
          RAG1(1) = JX1 - LCOGX(1)
          RAG1(2) = JY1 - LCOGY(1)
          RAG1(3) = JZ1 - LCOGZ(1)
          VECTA(1) = W1(1)
          VECTA(2) = W1(2)
          VECTA(3) = W1(3)
          CALL CPROD (VECTA,RAG1,MIC1,MJC1,MKC1)
          VECTB(1) = MIC1
          VECTB(2) = MJC1
          VECTB(3) = MKC1
          CALL CPROD (VECTA,VECTB,MIC1,MJC1,MKC1)

*      W2 X (W2 X RB/G2)
          RBG2(1) = JX1 - LCOGX(2)
          RBG2(2) = JY1 - LCOGY(2)
          RBG2(3) = JZ1 - LCOGZ(2)
          VECTA(1) = W2(1)
          VECTA(2) = W2(2)
          VECTA(3) = W2(3)
          CALL CPROD (VECTA,RBG2,MIC2,MJC2,MKC2)
          VECTB(1) = MIC2
          VECTB(2) = MJC2
          VECTB(3) = MKC2
          CALL CPROD (VECTA,VECTB,MIC2,MJC2,MKC2)

*      W2 X (W2 X RA/G2)
          RAG2(1) = JX2 - LCOGX(2)
          RAG2(2) = JY2 - LCOGY(2)
          RAG2(3) = JZ2 - LCOGZ(2)

```

```

      VECTA(1) = W2(1)
      VECTA(2) = W2(2)
      VECTA(3) = W2(3)
      CALL CPROD (VECTA,RAG2,MIC3,MJC3,MKC3)
      VECTB(1) = MIC3
      VECTB(2) = MJC3
      VECTB(3) = MKC3
      CALL CPROD(VECTA,VECTB,MIC3,MJC3,MKC3)
* W3 X (W3 X RB/G3)
      RBG3(1) = JX2 - LCOGX(3)
      RBG3(2) = JY2 - LCOGY(3)
      RBG3(3) = JZ2 - LCOGZ(3)
      VECTA(1) = W3(1)
      VECTA(2) = W3(2)
      VECTA(3) = W3(3)
      CALL CPROD (VECTA,RBG3,MIC4,MJC4,MKC4)
      VECTB(1) = MIC4
      VECTB(2) = MJC4
      VECTB(3) = MKC4
      CALL CPROD (VECTA,VECTB,MIC4,MJC4,MKC4)

* INERTIA TRANSFORMATION
      DO 390 I = 2, 3
      TMAT(2,1) = -DCOS ( THZ )
      TMAT(2,2) = -DSIN ( THZ )
      TMAT(2,3) = 0.0D0
      TMAT(3,1) = DRCSX(I)
      TMAT(3,2) = DRCSY(I)
      TMAT(3,3) = DRCSZ(I)

      VECTA(1) = TMAT(2,1)
      VECTA(2) = TMAT(2,2)
      VECTA(3) = TMAT(2,3)
      VECTB(1) = TMAT(3,1)
      VECTB(2) = TMAT(3,2)
      VECTB(3) = TMAT(3,3)
      CALL CPROD (VECTA,VECTB,MI1,MJ1,MK1)
      TMAT(1,1) = MI1
      TMAT(1,2) = MJ1
      TMAT(1,3) = MK1
      TMATTR(1,1) = TMAT(1,1)
      TMATTR(1,2) = TMAT(2,1)
      TMATTR(1,3) = TMAT(3,1)
      TMATTR(2,1) = TMAT(1,2)
      TMATTR(2,2) = TMAT(2,2)
      TMATTR(2,3) = TMAT(3,2)
      TMATTR(3,1) = TMAT(1,3)
      TMATTR(3,2) = TMAT(2,3)
      TMATTR(3,3) = TMAT(3,3)

      DO 375 I1 = 1,3
      DO 375 J = 1,3
      TEMP = 0.0D0
      DO 370 K = 1,3
      TEMP = TMATTR(I1,K) * LIMAT(I,K,J) + TEMP
370 CONTINUE

```



```

375      MATTMP(I1,J) = TEMP
        CONTINUE

        DO 380 I1 = 1,3
        DO 380 J = 1,3
            TEMP = 0.0D0
            DO 378 K = 1,3
                TEMP = MATTMP(I1,K) * TMAT(K,J) + TEMP
378      CONTINUE
        NIMAT(I1,J) = TEMP
380      CONTINUE

```

```

        IXXT(I) = NIMAT(1,1)
        IXYT(I) = NIMAT(1,2)
        IXZT(I) = NIMAT(1,3)
        IYYT(I) = NIMAT(2,2)
        IYZT(I) = NIMAT(2,3)
        IZZT(I) = NIMAT(3,3)

```

```

390      CONTINUE

```

```

***** ENTER CONSTANTS INTO MATRIX A *****

```

```

**      LINK ONE
*      SUM OF FORCES
*      X DIRECTION
395      MATA(1,1) = 1.0D0
        MATA(1,4) = M1
        MATA(1,10) = -1.0D0
        MATB(1) = 0.0D0
*      Y DIRECTION
        MATA(2,2) = 1.0D0
        MATA(2,5) = M1
        MATA(2,11) = -1.0D0
        MATB(2) = 0.0D0
*      Z DIRECTION
        MATA(3,3) = 1.0D0
        MATA(3,6) = M1
        MATA(3,12) = -1.0D0
        MATB(3) = -WG1
*      EQUATIONS AT JOINT ZERO
*      X DIRECTION
        MATA(4,4) = 1.0D0
        MATA(4,8) = RBG1(3)
        MATA(4,9) = -RBG1(2)
        MATB(4) = AX0 - MICO
*      Y DIRECTION
        MATA(5,5) = 1.0D0
        MATA(5,7) = -RBG1(3)
        MATA(5,9) = RBG1(1)
        MATB(5) = AYO - MJCO
*      Z DIRECTION
        MATA(6,6) = 1.0D0
        MATA(6,7) = RBG1(2)
        MATA(6,8) = -RBG1(1)

```

```

      MATB(6) = AZO - MKCO
* SUM OF MOMENTS EQUATIONS
* X DIRECTION
      MATA(7,2) = RBG1(3)
      MATA(7,3) = -RBG1(2)
      MATA(7,7) = -IXXT(1)
      MATA(7,8) = IXYT(1)
      MATA(7,9) = IXZT(1)
      MATA(7,11) = -RAG1(3)
      MATA(7,12) = RAG1(2)
      MATB(7) = T1X - TOX
* Y DIRECTION
      MATA(8,1) = -RBG1(3)
      MATA(8,3) = RBG1(1)
      MATA(8,7) = IXYT(1)
      MATA(8,8) = -IYYT(1)
      MATA(8,9) = IYZT(1)
      MATA(8,10) = RAG1(3)
      MATA(8,12) = -RAG1(1)
      MATB(8) = T1Y - TOY
* Z DIRECTION
      MATA(9,1) = RBG1(2)
      MATA(9,2) = -RBG1(1)
      MATA(9,7) = IXZT(1)
      MATA(9,8) = IYZT(1)
      MATA(9,9) = -IZZT(1)
      MATA(9,10) = -RAG1(2)
      MATA(9,11) = RAG1(1)
      MATB(9) = T1Z - TOZ

** LINK TWO
* SUM OF FORCES
* X DIRECTION
460      MATA(10,10) = 1.0D0
      MATA(10,13) = M2
      MATA(10,19) = -1.0D0
      MATB(10) = 0.0D0
* Y DIRECTION
      MATA(11,11) = 1.0D0
      MATA(11,14) = M2
      MATA(11,20) = -1.0D0
      MATB(11) = 0.0D0
* Z DIRECTION
      MATA(12,12) = 1.0D0
      MATA(12,15) = M2
      MATA(12,21) = -1.0D0
      MATB(12) = -WG2
* EQUATIONS AT JOINT ONE
* X DIRECTION
      MATA(13,4) = -1.0D0
      MATA(13,8) = -RAG1(3)
      MATA(13,9) = RAG1(2)
      MATA(13,13) = 1.0D0
      MATA(13,17) = RBG2(3)
      MATA(13,18) = -RBG2(2)

```

```

*      MATB(13)      =  MIC1 - MIC2
Y DIRECTION
  MATA(14,5)  = -1.0D0
  MATA(14,7)  =  RAG1(3)
  MATA(14,9)  = -RAG1(1)
  MATA(14,14) =  1.0D0
  MATA(14,16) = -RBG2(3)
  MATA(14,18) =  RBG2(1)
  MATB(14)    =  MJC1 - MJC2
*      Z DIRECTION
  MATA(15,6)  = -1.0D0
  MATA(15,7)  = -RAG1(2)
  MATA(15,8)  =  RAG1(1)
  MATA(15,15) =  1.0D0
  MATA(15,16) =  RBG2(2)
  MATA(15,17) = -RBG2(1)
  MATB(15)    =  MKC1 - MKC2
*      SUM OF MOMENTS EQUATIONS
*      X DIRECTION
  MATA(16,11) =  RBG2(3)
  MATA(16,12) = -RBG2(2)
  MATA(16,16) = -IXXT(2)
  MATA(16,17) =  IXYT(2)
  MATA(16,18) =  IXZT(2)
  MATA(16,20) = -RAG2(3)
  MATA(16,21) =  RAG2(2)
  MATB(16)    =  T2X - T1X
*      Y DIRECTION
  MATA(17,10) = -RBG2(3)
  MATA(17,12) =  RBG2(1)
  MATA(17,16) =  IXYT(2)
  MATA(17,17) = -IYYT(2)
  MATA(17,18) =  IYZT(2)
  MATA(17,19) =  RAG2(3)
  MATA(17,21) = -RAG2(1)
  MATB(17)    =  T2Y - T1Y
*      Z DIRECTION
  MATA(18,10) =  RBG2(2)
  MATA(18,11) = -RBG2(1)
  MATA(18,16) =  IXZT(2)
  MATA(18,17) =  IYZT(2)
  MATA(18,18) = -IZZT(2)
  MATA(18,19) = -RAG2(2)
  MATA(18,20) =  RAG2(1)
  MATB(18)    =  T2Z - T1Z

**     LINK THREE
*      SUM OF FORCES
*      X DIRECTION
530    MATA(19,19) =  1.0D0
        MATA(19,22) =  M3
        MATB(19)   =  0.0D0
*      Y DIRECTION
        MATA(20,20) =  1.0D0
        MATA(20,23) =  M3

```

```

      MATB(20)      =  0.0D0
*  Z DIRECTION
      MATA(21,21) =  1.0D0
      MATA(21,24) =  M3
      MATB(21) = -WG3
*  EQUATIONS AT JOINT TWO
*  X DIRECTION
      MATA(22,13) = -1.0D0
      MATA(22,17) = -RAG2(3)
      MATA(22,18) =  RAG2(2)
      MATA(22,22) =  1.0D0
      MATA(22,26) =  RBG3(3)
      MATA(22,27) = -RBG3(2)
      MATB(22)      =  MIC3 - MIC4
*  Y DIRECTION
      MATA(23,14) = -1.0D0
      MATA(23,16) =  RAG2(3)
      MATA(23,18) = -RAG2(1)
      MATA(23,23) =  1.0D0
      MATA(23,25) = -RBG3(3)
      MATA(23,27) =  RBG3(1)
      MATB(23)      =  MJC3 - MJC4
*  Z DIRECTION
      MATA(24,15) = -1.0D0
      MATA(24,16) = -RAG2(2)
      MATA(24,17) =  RAG2(1)
      MATA(24,24) =  1.0D0
      MATA(24,25) =  RBG3(2)
      MATA(24,26) = -RBG3(1)
      MATB(24)      =  MKC3 - MKC4
*  SUM OF MOMENTS EQUATIONS
*  X DIRECTION
      MATA(25,20) =  RBG3(3)
      MATA(25,21) = -RBG3(2)
      MATA(25,25) = -IXXT(3)
      MATA(25,26) =  IXYT(3)
      MATA(25,27) =  IXZT(3)
      MATB(25)      = -T2X
*  Y DIRECTION
      MATA(26,19) = -RBG3(3)
      MATA(26,21) =  RBG3(1)
      MATA(26,25) =  IXYT(3)
      MATA(26,26) = -IYYT(3)
      MATA(26,27) =  IYZT(3)
      MATB(26)      = -T2Y
*  Z DIRECTION
      MATA(27,19) =  RBG3(2)
      MATA(27,20) = -RBG3(1)
      MATA(27,25) =  IXZT(3)
      MATA(27,26) =  IYZT(3)
      MATA(27,27) = -IZZT(3)
      MATB(27)      = -T2Z

***** FIRST LINK IS CONSTRAINED NOT TO ROTATE *****
      DO 600 I = 4,9
        DO 595 J = 1,27

```

```

        MATA(I,J) = 0.0D0
        MATA(J,I) = 0.0D0
595    CONTINUE
        MATB(I) = 0.0D0
600    CONTINUE
        MATA(4,4) = 1.0D0
        MATA(5,5) = 1.0D0
        MATA(6,6) = 1.0D0
        MATA(7,7) = 1.0D0
        MATA(8,8) = 1.0D0
        MATA(9,9) = 1.0D0
        DO 602 I = 13,17
            DO 597 J = 1,27
                MATA(I,J) = 0.0D0
                MATA(J,I) = 0.0D0
597    CONTINUE
                MATB(I) = 0.0D0
602    CONTINUE
                MATA(13,13) = 1.0D0
                MATA(14,14) = 1.0D0
                MATA(15,15) = 1.0D0
                MATA(16,16) = 1.0D0
                MATA(17,17) = 1.0D0
605    DO 610 J = 1,27
                MATA(18,J) = 0.0D0
                MATA(27,J) = 0.0D0
610    CONTINUE

        MATA(18,9) = -1.0D0
        MATA(18,18) = 1.0D0
        MATB(18) = 0.0D0
        MATA(27,9) = -1.0D0
        MATA(27,27) = 1.0D0
        MATB(27) = 0.0D0

*      CALL EQUATION SOLVER PROGRAM FROM IMSL
620    CALL LEQT2F(MATA,M,N,IA,MATB,IDGT,WKAREA,IER)
        IF (IER.EQ.0) THEN
            GOTO 640
        ELSE
            WRITE (*,624) IER
624    FORMAT (I5)
            DO 635 I = 1, 27
                WRITE (*,627) I
627    FORMAT (I7)
                DO 631 J = 1, 27, 3
                    WRITE (*,630) J,MATA(I,J),J+1,MATA(I,J+1),J+2,MATA(I,J+2)
630    FORMAT (I5,F13.5,I5,F13.5,I5,F13.5)
631    CONTINUE
                    WRITE (*,633) I,MATB(I)
633    FORMAT (I3,F15.5)
635    CONTINUE
                CALL ENDJOB
            END IF

```



```

***      FIND LCOGX, LCOGY, LCOGZ, THETA VALUES, WX, WY, WZ
***      MODIFIED BY R. M. VERBOS

*      JOINT ZERO
640      FX0 = MATB(1)
          FY0 = MATB(2)
          FZ0 = MATB(3)

*      LINK ONE
*      SINCE LINK1 IS CONSTRAIN TO ROTATE AROUND THE Z ONLY
          AX1 = 0.0D0
          AY1 = 0.0D0
          AZ1 = 0.0D0
*660      AX1      = MATB(4)
*          VELX1    = INTGRL(0., AX1)
*          LCOGX1   = INTGRL(X1, VELX1)
*          LCOGX(1) = LCOGX1
*          AY1      = MATB(5)
*          VELY1    = INTGRL(0., AY1)
*          LCOGY1   = INTGRL(Y1, VELY1)
*          LCOGY(1) = LCOGY1
*          AZ1      = MATB(6)
*          VELZ1    = INTGRL(0., AZ1)
*          LCOGZ1   = INTGRL(Z1, VELZ1)
*          LCOGZ(1) = LCOGZ1
          WD1X      = MATB(7)
          W1X       = INTGRL(0., WD1X)
          WDX(1)    = WD1X
          W1(1)     = W1X
          WD1Y      = MATB(8)
          W1Y       = INTGRL(0., WD1Y)
          WDY(1)    = WD1Y
          W1(2)     = W1Y
          WD1Z      = MATB(9)
          W1Z       = INTGRL(0., WD1Z)
          WDZ(1)    = WD1Z
          W1(3)     = W1Z
***      ADDED BY R. M. VERBOS
685      THZ       = INTGRL(0., W1Z)
          THZANG    = THZ * RADEG
          COSTHZ    = DCOS(THZ)
          SINTHZ    = DSIN(THZ)

*      IF THE 1ST LINK IS CONSTRAIN TO ROTATE IN THE Z DIRECTION ONLY
*      THE DIRECTION COSINE AND DIRECTION COSINE ANGLES ARE CONSTANT
*      AND DO NOT NEED TO BE CALCULATED
*
*      CALC DIRECTIONAL COSINES FOR LINK ONE
*
*695      LNCOG1 = DSQRT ( LCOGX1*LCOGX1 + LCOGY1*LCOGY1 + ...
*                      LCOGZ1*LCOGZ1 )
*700      DRCSX(1) = LCOGX1 / LNCOG1
*          DRCSY(1) = LCOGY1 / LNCOG1
*          DRCSZ(1) = LCOGZ1 / LNCOG1
*          AMP = DSQRT(DRCSX(1)*DRCSX(1)+DRCSY(1)*DRCSY(1)+...
*                  DRCSZ(1)*DRCSZ(1))

```

```

*      DRCSX(1) = DRCSX(1)/AMP
*      DRCSY(1) = DRCSY(1)/AMP
*      DRCSZ(1) = DRCSZ(1)/AMP
*720   DRCANX(1) = DACOS(DRCSX(1)) * RADEG
*      DRCANY(1) = DACOS(DRCSY(1)) * RADEG
*      DRCANZ(1) = DACOS(DRCSZ(1)) * RADEG
*
**     JOINT LOCATION
*730   JX1 = LINKL1 * DRCSX(1)
*      JY1 = LINKL1 * DRCSY(1)
*      JZ1 = LINKL1 * DRCSZ(1)

*     JOINT ONE
735   FX1 = MATB(10)
      FY1 = MATB(11)
      FZ1 = MATB(12)

      AX2 = 0.0D0
      AY2 = 0.0D0
      AZ2 = 0.0D0

**     LINK TWO
740   AX2 = MATB(13)
*      VELX2 = INTGRL(0.,AX2)
*      LCOGX2 = INTGRL(X2,VELX2)
*      LCOGX(2) = LCOGX2
*      AY2 = MATB(14)
*      VELY2 = INTGRL(0.,AY2)
*      LCOGY2 = INTGRL(Y2,VELY2)
*      LCOGY(2) = LCOGY2
*      AZ2 = MATB(15)
*      VELZ2 = INTGRL(0.,AZ2)
*      LCOGZ2 = INTGRL(Z2,VELZ2)
*      LCOGZ(2) = LCOGZ2
*      WD2X = MATB(16)
*      W2X = INTGRL(0.,WD2X)
*      WDX(2) = WD2X
*      W2(1) = W2X
*      WD2Y = MATB(17)
*      W2Y = INTGRL(0.,WD2Y)
*      WDY(2) = WD2Y
*      W2(2) = W2Y
*      WD2Z = MATB(18)
*      W2Z = INTGRL(0.,WD2Z)
*      WDZ(2) = WD2Z
*      W2(3) = W2Z

*     GET THE DIRECTION COSINES FOR THE LINK TWO
*      DRCSX(2) = (LCOGX2 - JX1) / LNCOG2
*      DRCSY(2) = (LCOGY2 - JY1) / LNCOG2
*      DRCSZ(2) = (LCOGZ2 - JZ1) / LNCOG2
*90   AMP = DSQRT(DRCSX(2)*DRCSX(2)+DRCSY(2)*DRCSY(2)+...
*      DRCSZ(2)*DRCSZ(2))
*      DRCSX(2) = DRCSX(2)/AMP
*      DRCSY(2) = DRCSY(2)/AMP
*      DRCSZ(2) = DRCSZ(2)/AMP

```

```

*      DRCANX(2) = DACOS(DRCSX(2)) * RADEG
*      DRCANY(2) = DACOS(DRCSY(2)) * RADEG
*      DRCANZ(2) = DACOS(DRCSZ(2)) * RADEG

```

```

*      JOINT LOCATION

```

```

800      JX2 = JX1 + LINKL2 * DRCSX(2)
          JY2 = JY1 + LINKL2 * DRCSY(2)
          JZ2 = JZ1 + LINKL2 * DRCSZ(2)

```

```

*      JOINT TWO

```

```

805      FX2 = MATB(19)
          FY2 = MATB(20)
          FZ2 = MATB(21)

```

```

**      LINK THREE

```

```

812      AX3      = MATB(22)
          VELX3    = INTGRL(0.,AX3)
          LCOGX3   = INTGRL(X3,VELX3)
          LCOGX(3) = LCOGX3
          AY3      = MATB(23)
          VELY3    = INTGRL(V3IC,AY3)
          LCOGY3   = INTGRL(Y3,VELY3)
          LCOGY(3) = LCOGY3
          AZ3      = MATB(24)
          VELZ3    = INTGRL(0.,AZ3)
          LCOGZ3   = INTGRL(Z3,VELZ3)
          LCOGZ(3) = LCOGZ3
          WD3X     = MATB(25)
          W3X      = INTGRL(W3IC,WD3X)
          WDX(3)   = WD3X
          W3(1)    = W3X
          WD3Y     = MATB(26)
          W3Y      = INTGRL(0.,WD3Y)
          WDY(3)   = WD3Y
          W3(2)    = W3Y
          WD3Z     = MATB(27)
          W3Z      = INTGRL(0.,WD3Z)
          WDZ(3)   = WD3Z
          W3(3)    = W3Z

```

```

*      CALC DIRECTIONAL COSINES FOR LINK THREE

```

```

845      DRCSX(3) = (LCOGX3 - JX2) / LNCOG3
          DRCSY(3) = (LCOGY3 - JY2) / LNCOG3
          DRCSZ(3) = (LCOGZ3 - JZ2) / LNCOG3
865      AMP = DSQRT(DRCSX(3)*DRCSX(3)+DRCSY(3)*DRCSY(3)+...
                  DRCSZ(3)*DRCSZ(3))
          DRCSX(3) = DRCSX(3)/AMP
          DRCSY(3) = DRCSY(3)/AMP
          DRCSZ(3) = DRCSZ(3)/AMP
          DRCANX(3) = DACOS(DRCSX(3)) * RADEG
          DRCANY(3) = DACOS(DRCSY(3)) * RADEG
          DRCANZ(3) = DACOS(DRCSZ(3)) * RADEG

```

```

*      TIP LOCATION

```

```

875      TIPX = JX2 + LINKL3 * DRCSX(3)
          TIPY = JY2 + LINKL3 * DRCSY(3)

```

```

      TIPZ = JZ2 + LINKL3 * DRCSZ(3)

*      FIND THE ANGLE BETWEEN THE LIMKS
880    ANG23X = DRCANX(2) - DRCANX(3)
      ANG23Y = DRCANY(2) - DRCANY(3)
      ANG23Z = DRCANZ(2) - DRCANZ(3)
      ANG12X = DRCANX(1) - DRCANX(2)
      ANG12Y = DRCANY(1) - DRCANY(2)
      ANG12Z = DRCANZ(1) - DRCANZ(2)

DYNAMIC
NOSORT
***** INPUT TORQUE AT JOINTS
      T2FNC = (13.5 * SIN (PI / (30.0) * TIME)) * RCNFAC
      T2 = T2FNC
      T2X = T2

END
STOP

FORTRAN

*      SUBROUTINE TO COMPUTE THE CROSS PRODUCT OF TWO VECTORS

      SUBROUTINE CPROD(VECTA,VECTB,MI,MJ,MK)
940    IMPLICIT REAL*8 (A-Z)
      DIMENSION VECTA(3),VECTB(3)
      MI = VECTA(2) * VECTB(3) - VECTA(3) * VECTB(2)
      MJ = VECTA(3) * VECTB(1) - VECTA(1) * VECTB(3)
      MK = VECTA(1) * VECTB(2) - VECTA(2) * VECTB(1)
      RETURN
      END

```

## APPENDIX E

### BASIC OPERATING INSTRUCTIONS FOR THE NAVAL POSTGRADUATE SCHOOL RIGID MANIPULATOR TEST BED

#### A. SYSTEM FAMILIARIZATION

Before operating the manipulator test bed, you should ensure you are familiar with its components and basic procedures of operation. If you are already familiar with the components of the manipulator and their set-up, you will need to review this section to operate the hardware.

The test bed consists of four major sub-systems: the Neptune II hydraulically operated manipulator; an IBM PC-AT used to control the manipulator and gather data (at the present time); an IBM PC-XT (not in the system at present); and an Electronic Control Panel used in switching, data gathering, and interfacing.

##### 1. The Manipulator

The manipulator (Figure E-1), being a hydraulic mechanism, is subject to leakage. It is advisable to wear old clothing while in the lab. The operating pressure of the system is around 100 psi and a minor leak may spray anywhere in the lab.

The source of the hydraulic fluid is the pump cabinet against the forward side of the wall in front of the computers. This cabinet contains a pump which is operated from the ECP, a reservoir which is underneath the pump, and an accumulator that is alongside of the pump (Figure E-2).



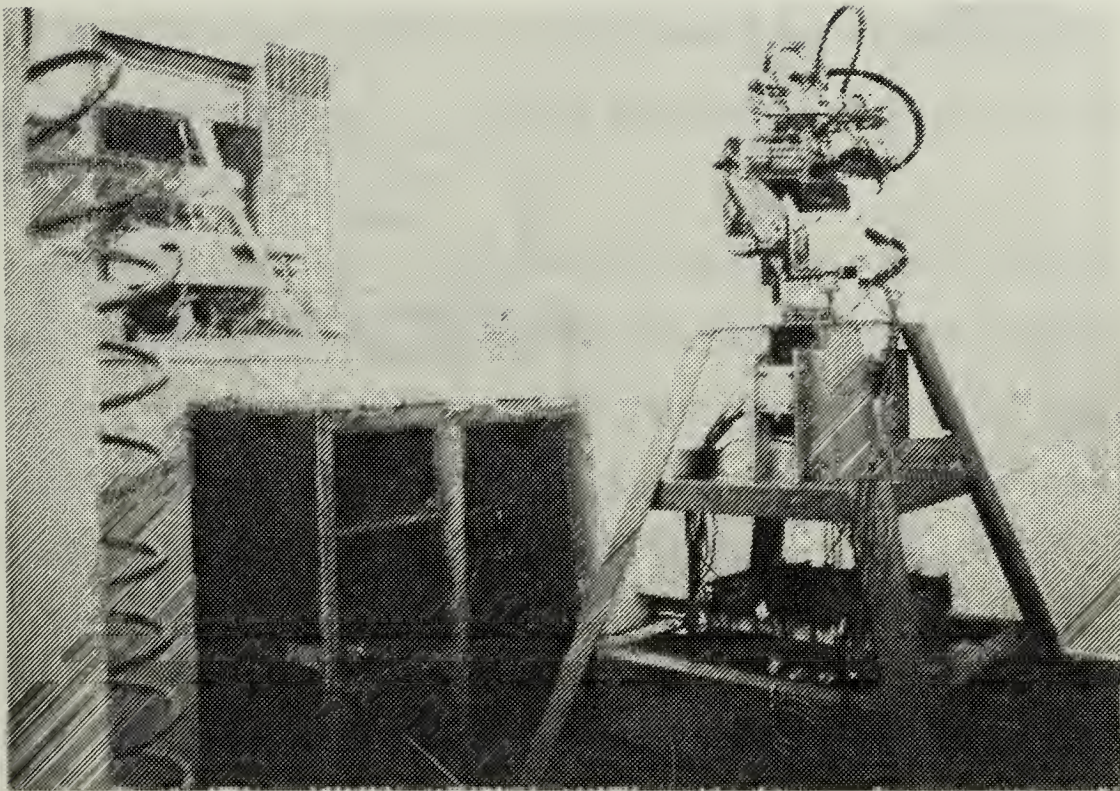


Figure E-1. **Manipulator**

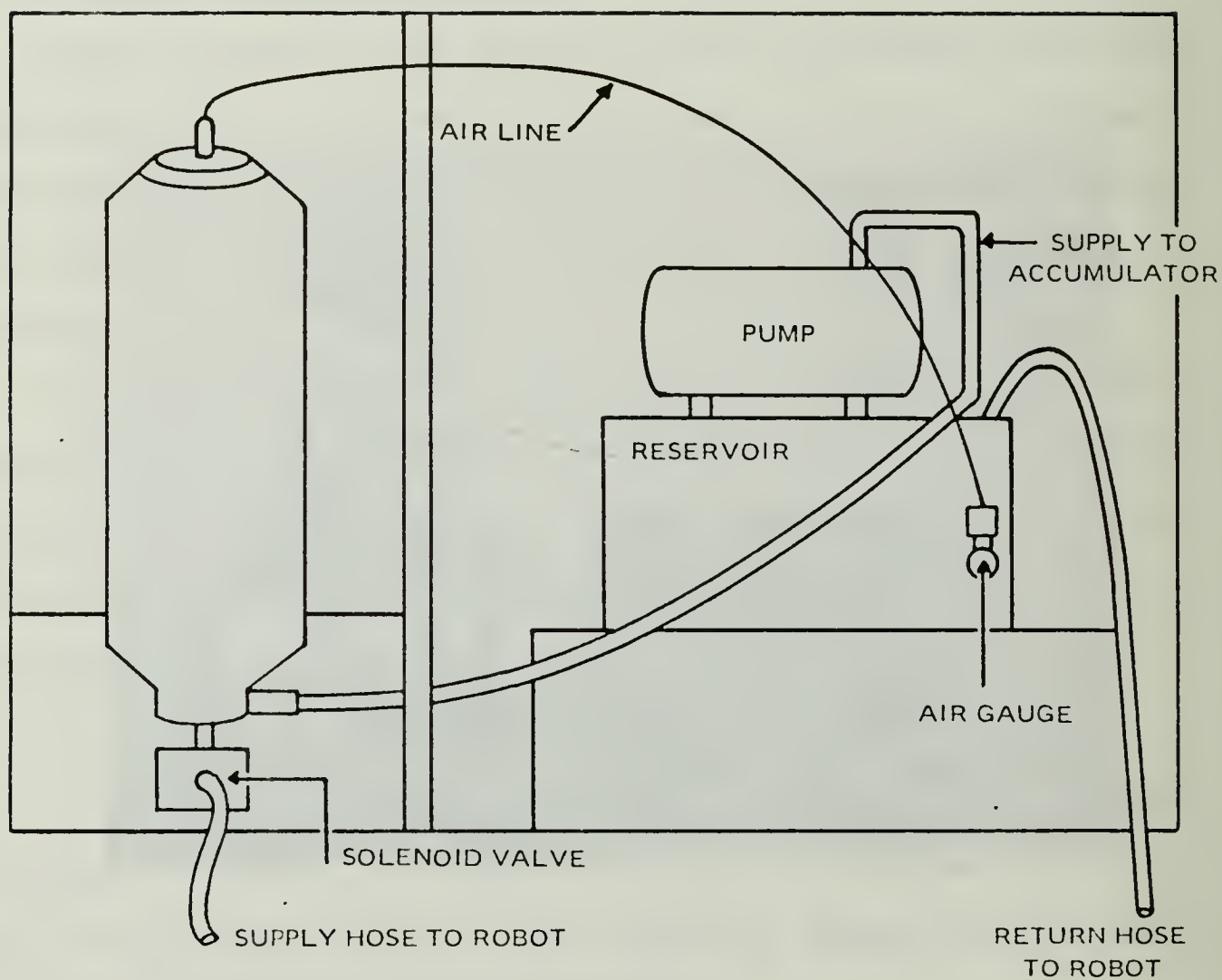


Figure E-2. **Pump Cabinet**

The reservoir has a fill hole on the top. It is necessary to check this before operating the system and ensure that the level is around half when the system is not in operation. You will also notice a pressure gage which reads the pressure in the accumulator. This pressure should be between 80 and 100 psi and may be increased by pressurizing the bladder in the accumulator to a higher pressure.

If you look closely at the accumulator, you will notice a solenoid valve at the outlet. This valve has been installed because of previous problems and will cut off all hydraulic flow to the manipulator when it is shut from the ECP.

Underneath the manipulator itself is a set of Electronic Printed Circuit Boards (EPCB). These boards control the opening and shutting of the solenoid valves to the joint pistons (see Ref. 9 for more information). The important thing to know about these boards is that you should avoid getting hydraulic fluid on them.

## **2. The Computers**

At present, the IBM-AT is the only computer hooked up in the system. It is easy to follow because it has been set up with a menu-based operating system. The power to the AT is under the right side of the computer stand. The first thing that will happen when you turn the power on is that the AT will go thru a cold-start boot-up, which may take a minute or two. You will then notice a listing of subdirectories on the screen. Most of these subdirectories are of little use because the files are proprietary in nature and the author has long since transferred away from NPS, but feel free to skim through them.



For the initial running of the manipulator, you will want to select the BATCHES subdirectory. Once you are in this subdirectory, you will select the NEPTROLA file to run the manipulator. This file contains the control program for operating the manipulator in the solenoid mode. The program is a useful starting point for examining the range of the arm motion.

NEPTROLA is a menu-driven set-up that asks various questions to get the system in the proper operating configuration. The actual operation will be discussed in the operation section of this appendix.

### **3. Electronic Control Panel (ECP)**

The ECP is located between the IBM-AT and the IBM-XT and is the heart of the control of the manipulator. The ECP contains several interface boards which should not be tampered with without talking to Tom Cristian of the Mechanical Engineering technical staff. The front of the panel is divided into four sections (Figure E-3). Starting at the top, the first sub-panel consists of two sets of electrical jacks. The first six from the left are connected to the position data of the interface boards and will provide joint 0 thru 5 position output. The last two jacks are for servo operation which is no longer connected (more information on servo operation is available in Ref. 13). The next panel down was for control of the servovalves; again, more information is available in Reference 13. The next panel contains a power switch which selects power to the panel and electrical jacks to

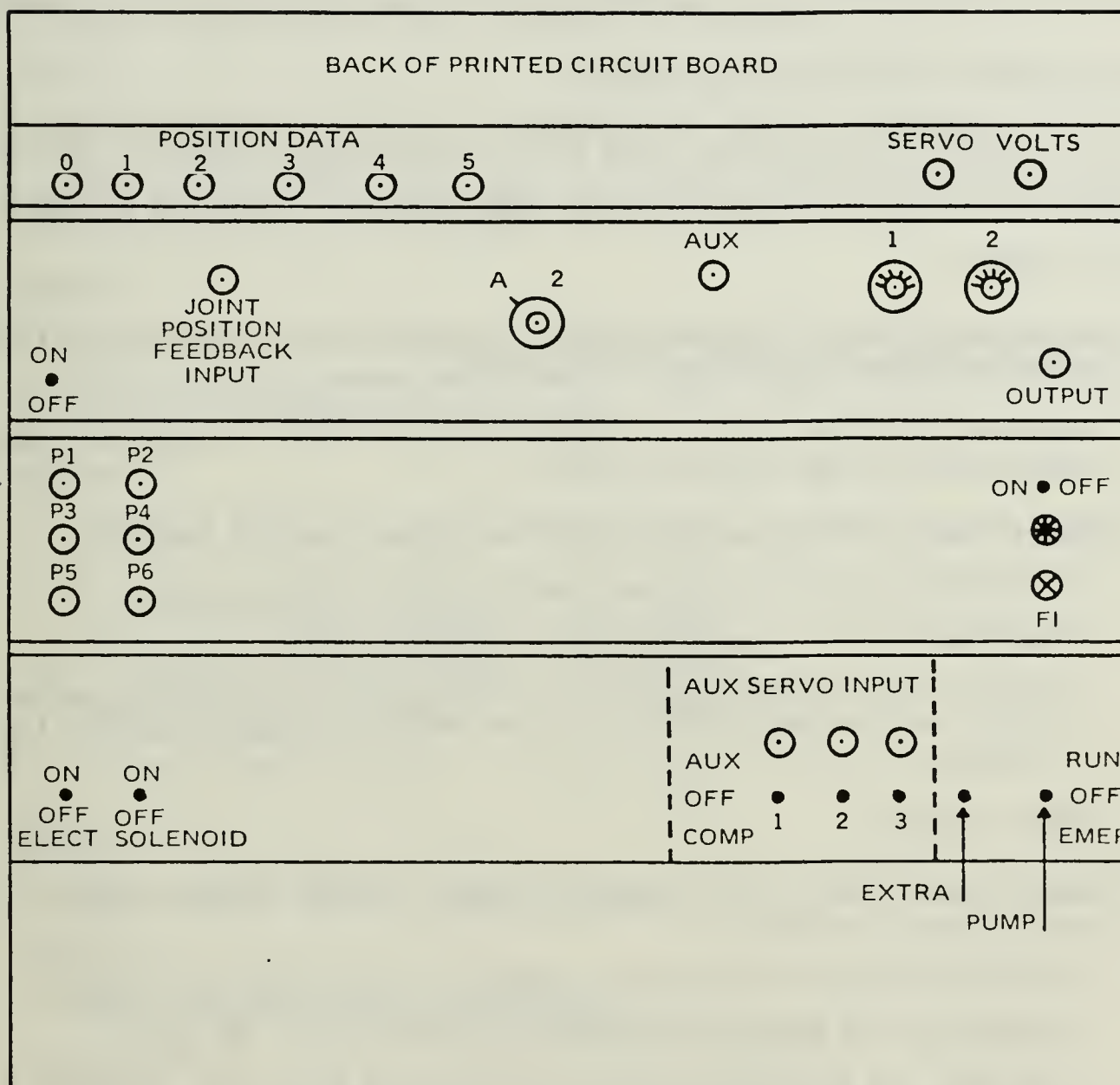


Figure E-3. Electronic Control Panel (ECP)



monitor the pressure transducer output for torque calculations. Pressure transducers have been installed on joints 1 and 2. The electrical jacks are wired so that P1 and P2 are connected to joint 1 transducers, P3 and P4 are connected to joint 2 transducers, and P5 and P6 are available for further connections.

The bottom panel is the power panel and consists of seven switches and three electrical jacks. The switches, from left to right, are as follows:

- Electronic Control Power, which supplies the electricity to the interface board at the back of the panel.
- Solenoid Current Switch, which must be “ON” to operate the manipulator in the solenoid mode.
- Servo Master Switches (3), which are three-position switches:
  - (DOWN) is for selecting servovalve control from the AT
  - (MIDDLE) is for operating in the solenoid mode
  - (UP) is for for operating the servovalves from an auxiliary signal which is connected to the three electrical jacks above the switches.
- Extra switch.
- (Rightmost) Pump Power/Solenoid Valve Control Switch. This is a three-position switch:
  - (UP) the pump is in the run mode and controlled by pressure
  - (MIDDLE) the pump is off and the solenoid valve is open
  - (DOWN) the pump is off and the solenoid valve has power to it and is shut (this is an emergency cut-off position).

## **B. OPERATING THE ARM**

### **1. The Solenoid Mode**

Following is a step-by-step procedure for operating the Neptune II manipulator in the solenoid mode:

#### **STEP 1**

Check the accumulator to ensure that there is sufficient fluid to operate the manipulator. If the level is low, add a 50/50 mix of Hydro Lube and water until the level is between one-half and three-quarters (DO NOT FILL).

#### **STEP 2**

Check the accumulator pressure to ensure it is between 80 and 100 psi. If the pressure is low or high, bleed or add air through the hose connection as necessary.

#### **STEP 3**

From the ECP turn the following switches to the position referred to:

- Electronic Power Switch      ON (UP)
- Solenoid Current Switch      ON (UP)
- Servo Master Switches      OFF (MIDDLE)
- Pump Power Switch      ON (UP)

#### **STEP 4**

Now turn the IBM-AT computer on.

#### **STEP 5**

Select the BATCHES subdirectory.

#### **STEP 6**

Select the NEPTROLA program and answer "yes" (Y) to the question whether you want to operate without ADC. This will allow you to operate from the keyboard. The next question will give several options—pick the keyboard option. Follow the given computer menu and vary the joint positions as desired.

## STEP 7

To leave the NEPTROLA program, hit ESC, then hit the control and break key at the same time. After this, type "SYSTEM" and you will be returned to the BATCHES subdirectory menu.

### **2. Servovalve Operation**

At present, the servo valves have been removed from the system but the software and wiring are still available. When the servo-valves are reinstalled, refer to [Ref. 13] for operating procedures.

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